January 28

SWBAT:

Use differentiation and integration to find power series

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots + x^n + \ldots = \sum_{n=0}^{\infty} x^n \qquad -1 < x < 1$$

$$\cos x \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots + (-1)^n \frac{x^{2n}}{(2n)!} + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots + \frac{x^n}{n!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\frac{1}{3-x} \qquad \frac{1}{1-x}$$

$$= \frac{1}{1+(2-x)} = \frac{1}{1-(-(2-x))}$$

$$= \frac{1}{1-(x-2)} \qquad r = x-2$$

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find the power series expansion for $\frac{1}{(1-x)^2}$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots = \sum_{n=0}^{\infty} x^n$$

$$\frac{1}{(1-x)^2} \qquad \frac{x^n}{(1-x)^{n-1}} \qquad (x^n)^2$$

$$\frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{0-(-1)}{(1-x)^2} = \frac{1}{(1-x)^2}$$

$$\frac{d}{dx}\left(\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots = \sum_{n=0}^{\infty} x^n\right)$$

$$\frac{1}{(1-x)^2} = 0 + 1 + 2x + 3x^2 + \cdots + nx^{n-1} + \cdots = \sum_{n=1}^{\infty} nx^{n-1}$$

IoC: $-1 < x < 1$

Evaluate $\sum_{n=1}^{\infty} \frac{n}{3^n}$

$$= \sum_{n=1}^{\infty} n\left(\tfrac{1}{3}\right)^n \xrightarrow{x=\frac{1}{3}} \sum_{n=1}^{\infty} n\left(\tfrac{1}{3}\right)^1\left(\tfrac{1}{3}\right)^{n-1} = \frac{1}{3}\sum_{n=1}^{\infty} n\left(\tfrac{1}{3}\right)^{n-1}$$

$$= \frac{1}{3}\left(\frac{1}{\left(1-\left(\frac{1}{3}\right)\right)^2}\right) = \frac{1}{3}\left(\frac{1}{\frac{4}{9}}\right) = \frac{1}{3}\left(\frac{9}{4}\right) = \frac{9}{12} = \frac{3}{4}$$

find the p.s. expansion for $\frac{1}{1+x^2}$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots = \sum_{n=0}^{\infty} x^n$$

$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = 1 + (-x^2) + (-x^2)^2 + (-x^2)^3 + \cdots + (-x^2)^n + \cdots$$

$$= 1 - x^2 + x^4 - x^6 + \cdots + (-1)^n x^{2n} + \cdots = \sum_{n=0}^{\infty} (-1)^n x^{2n}$$

IoC: $|x^2| < 1$

$$-1 < x^2 < 1 \rightarrow \cancel{\pm\sqrt{-1} <} x$$

$$\searrow x < \pm\sqrt{1} \rightarrow -1 < x < 1$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + C$$

find the p.s. expansion for $\tan^{-1} x$

$$\int \left(1 - x^2 + x^4 - x^6 + \cdots + (-1)^n x^{2n} + \cdots\right) dx$$

$$= x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = \tan^{-1} x$$

IoC: $-1 < x < 1$