

January 30

SWBAT:

Find the power series
representation of a given
function

$$\begin{aligned}\frac{1}{1+x} &= 1 - x + x^2 - x^3 + \dots + (-1)^n x^n \\ \int \frac{1}{1+x} dx &= \int 1 - x + x^2 - x^3 + \dots + (-1)^n x^n dx \\ \ln(x+1) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^n \frac{x^{n+1}}{n+1} + \dots \\ &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}\end{aligned}$$

$$\frac{1}{4+3x} = \frac{1}{4(1+\frac{3x}{4})} = \frac{1}{4(1-\frac{-3x}{4})}$$

$$= \frac{1}{4} \left(1 + \frac{-3x}{4} + \left(\frac{-3x}{4}\right)^2 + \dots + (-1)^n \left(\frac{-3x}{4}\right)^n + \dots \right)$$

$$= \frac{1}{4} - \frac{3x}{4^2} + \frac{3^2 x^2}{4^3} + \dots + (-1)^n \frac{3^n x^n}{4^{n+1}} + \dots$$

$$-1 < \frac{-3x}{4} < 1$$

$$-4 < -3x < 4$$

$$\frac{4}{3} > x > -\frac{4}{3} \rightarrow -\frac{4}{3} < x < \frac{4}{3}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^{n+1}}{n+1} = (x-1) + \frac{-(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \dots + \frac{(-1)^n (x-1)^{n+1}}{n+1} + \dots = \ln x$$

$$\frac{d}{dx} \left(\sum a_n \right) = \boxed{1 - (x-1) + (x-1)^2 - (x-1)^3 + \dots + (-1)^n (x-1)^n + \dots = \frac{1}{x}}$$

$$\int_0^t x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \dots dx = \int_0^t \sin x \, dx$$

$$\frac{x^2}{2} - \frac{x^4}{4 \cdot 3!} + \frac{x^6}{6!} - \frac{x^8}{8!} + \dots + \frac{(-1)^{n+1} x^{2n}}{(2n)!} \Big|_0^t = -\cos x \Big|_0^t$$

$$\frac{t^2}{2!} - \frac{t^4}{4!} + \frac{t^6}{6!} - \frac{t^8}{8!} + \dots + \frac{(-1)^{n+1} t^{2n}}{(2n)!} = -\cos t - (-\cos 0)$$

$$-1 + \frac{t^2}{2!} - \frac{t^4}{4!} + \frac{t^6}{6!} - \frac{t^8}{8!} + \dots + \frac{(-1)^{n+1} t^{2n}}{(2n)!} = -\cos t$$

$$1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \dots + \frac{(-1)^n t^{2n}}{(2n)!} = \cos t$$

claurin
Series

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^n \left(\frac{x^{2n+1}}{(2n+1)!} \right) + \dots = \sum_{n=0}^{\infty} (-1)^n \left(\frac{x^{2n+1}}{(2n+1)!} \right)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^n \left(\frac{x^{2n}}{(2n)!} \right) + \dots = \sum_{n=0}^{\infty} (-1)^n \left(\frac{x^{2n}}{(2n)!} \right)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots + x^n + \dots = \sum_{n=0}^{\infty} x^n$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \dots + (-1)^n x^n + \dots = \sum_{n=0}^{\infty} (-1)^n x^n$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n-1} \frac{x^n}{n} + \dots = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$$

$$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots + (-1)^n \frac{x^{2n+1}}{2n+1} + \dots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

I.O.C
 $-\infty < x < \infty$

I.O.C
 $-1 < x < 1$

$$f(x) = \sin(2x)$$

write the
p.s. expansion

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \dots$$

$$\sin(2x) = 2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \dots + (-1)^n \frac{(2x)^{2n+1}}{(2n+1)!} + \dots$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{2n+1}}{(2n+1)!}$$

$$f(x) = 3x \tan^{-1}(x^2)$$

$$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots + (-1)^n \frac{x^{2n+1}}{2n+1} + \dots$$

$$\tan^{-1}(x^2) = x^2 - \frac{(x^2)^3}{3} + \frac{(x^2)^5}{5} - \dots + (-1)^n \frac{(x^2)^{2n+1}}{2n+1} + \dots$$

$$\tan^{-1}(x^2) = x^2 - \frac{x^6}{3} + \frac{x^{10}}{5} - \dots + (-1)^n \frac{x^{4n+2}}{2n+1} + \dots$$

$$3x \tan^{-1}(x^2) = 3x \left(x^2 - \frac{x^6}{3} + \frac{x^{10}}{5} - \dots + (-1)^n \frac{x^{4n+2}}{2n+1} + \dots \right)$$

$$= 3x^3 - x^7 + \frac{3x^{11}}{5} - \dots + (-1)^n \frac{3x^{4n+3}}{2n+1} + \dots$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{3x^{4n+3}}{2n+1}$$

Assignment #5

Pg 600 #1, 2, 3-27 (odd-skip #17, 23),
42, 63-66, 71, 72, 80-84