January 9

SWBAT:

Find the limit of a sequence

A sequence converges if

the sequence has a limit

if $\lim_{n \to \infty} a_n = L$ (where L is any number),

then the sequence converges to L

A sequence diverges if

the sequence does not have a limit

if $\lim_{n \to \infty} a_n$ DNE (or $= \pm\infty$),

then the sequence diverges

$a_n = \frac{n^2+3}{2n^2-1}$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^2+3}{2n^2-1} \Rightarrow \lim_{n\to\infty} \frac{n^2}{2n^2} = \frac{1}{2}$

Absolute Value Theorem

If $\lim_{n\to\infty} |a_n| = 0$

Then $\lim_{n\to\infty} a_n = 0$

If $\lim_{n\to\infty} |a_n| \neq 0$

then a_n diverges

(for alternating sequences)

$a_n = (-1)^n\left(\frac{5n+1}{n^2-2}\right)$

$\lim_{n\to\infty} a_n = 0$

$\lim_{n\to\infty} \left|(-1)^n\left(\frac{5n+1}{n^2-2}\right)\right| = \lim_{n\to\infty} \frac{5n+1}{n^2-2} = 0$

$a_n = (-1)^n\left(\frac{7n^2}{100n+\pi}\right)$

$\lim_{n\to\infty} a_n$ DNE, a_n diverges

$\lim_{n\to\infty} \left|(-1)^n\left(\frac{7n^2}{100n+\pi}\right)\right| = \lim_{n\to\infty} \frac{7n^2}{100n+\pi} = \infty$

$a_n = (-1)^n\left(\frac{6n^3-5}{20+2n^3}\right)$

$\lim_{n\to\infty} a_n$ DNE, a_n diverges

$\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{6n^3-5}{20+2n^3} = 3$

Series

sum of the terms in a sequence

$$a_1 + a_2 + a_3 + \cdots + a_n + \cdots = \sum_{n=1}^{\infty} a_n$$

expand the series

$$\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \cdots + \frac{1}{3^n} + \cdots$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = \frac{1}{1} + \frac{-1}{2} + \frac{1}{3} + \frac{-1}{4} + \cdots + \frac{(-1)^{n-1}}{n} + \cdots$$

Partial Sums

sum of a finite series

P_1 = sum of first term

P_2 = sum of the first two terms

$a_1 + a_2$

P_n = sum of the first n terms

$a_1 + a_2 + a_3 + \cdots + a_n$

$\sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n$ find the first five partial sums

$P_1 = \frac{1}{5}$ $= .2$

$P_2 = \frac{1}{5} + \frac{1}{25} = \frac{6}{25}$ $= .24$

$P_3 = \frac{1}{5} + \frac{1}{25} + \frac{1}{125} = \frac{31}{125}$ $= .248$

$P_4 = \frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \frac{1}{625} = \frac{156}{625}$ $= .2496$

$P_5 = \frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \frac{1}{625} + \frac{1}{3125} = \frac{781}{3125} = .24992$

.04

.008

.0016

Convergent Series	If the limit of the sequence of Partial Sums exists (limit = L) then the <u>series</u> converges to the limit L
Divergent Series	if the limit of the sequence of partial sums does not exist the the series diverges
If $\sum_{n=1}^{\infty} a_n$ converges	then $\lim_{n\to\infty} a_n = 0$ If the <u>series</u> converges then the <u>sequence</u> converges to 0.

n^{th} term test for divergence

If $\lim_{n\to\infty} a_n \neq 0$ then the series diverges