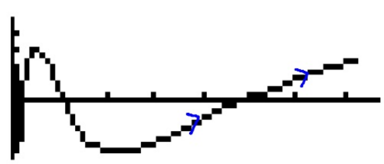
March 18

SWBAT:

Find the length of parametric curves and displacement and distance of vector defined functions

Graph the parametric function
$x(t) = e^t$
$y(t) = 3\cos(2t)$
for $0 < t < 6$



Find the slope of the tangent line when $t = 3$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-6\sin(2t)}{e^t}$$

at $t=3$

$$\frac{dy}{dx} = \frac{-6\sin(6)}{e^3} = .0835$$

slope

$$\left.\begin{aligned} x(3) &= e^3 \\ y(3) &= 3\cos(6) \end{aligned}\right\} \text{point}$$

$$y - 2.881 = .0835(x - 20.086)$$

$$y - 3\cos(6) = \frac{-6\sin 6}{e^3}(x - e^3)$$

How does arc length change if you have parametric functions?

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$L = \int_a^b \sqrt{\left(\frac{dx}{d\cancel{x}\,t}\right)^2 + \left(\frac{dy}{d\cancel{x}\,t}\right)^2}\; dt$$

Graph the parametric function
$x(t) = e^t$
$y(t) = 3\cos(2t)$
for $0 \le t \le 6$

Find the length of the curve on the given interval.

$0 \le t \le 6$
(bounds)

$\frac{dx}{dt} = e^t$

$\frac{dy}{dt} = -6\sin(2t)$

$$L = \int_0^6 \sqrt{(e^t)^2 + (-6\sin(2t))^2}\; dt$$

$$= 407.782$$

for vectors $\langle v_x(t), v_y(t) \rangle$

Displacement

$$\left\langle \int_a^b v_x(t)\,dt,\ \int_a^b v_y(t)\,dt \right\rangle$$

→ answer is a vector

Distance = magnitude of velocity

$$|\vec{v}(t)| = \int_a^b \sqrt{(\underbrace{v_x(t)}_{\substack{\text{velocity of } x \\ \text{(derivative of position} = \frac{dx}{dt})}})^2 + (\underbrace{v_y(t)}_{\substack{\text{velocity of } y \\ \text{(derivative of position} = \frac{dy}{dt})}})^2}\,dt = \text{Arc Length}$$

$\vec{r}(t) = \langle 2\cos(4\pi t), 4\sin(2\pi t) \rangle$

($\vec{r}$ = position)

find distance + displacement on $[0, \frac{3}{4}]$

$0 \le t \le \frac{3}{4}$

bounds

$\frac{dx}{dt} = v_x(t) = -8\pi\sin(4\pi t)$

$\frac{dy}{dt} = v_y(t) = 8\pi\cos(2\pi t)$

$$\text{Distance} = \int_0^{\frac{3}{4}} \sqrt{(-8\pi\sin(4\pi t))^2 + (8\pi\cos(2\pi t))^2}\,dt = 17.747$$

Displacement =

$$\left\langle \int_0^{3/4} -8\pi\sin(4\pi t)\,dt,\ \int_0^{3/4} 8\pi\cos(2\pi t)\,dt \right\rangle$$

$$= \langle -4, -4 \rangle$$