March 19

SWBAT:
Graph polar functions and
Find their derivatives

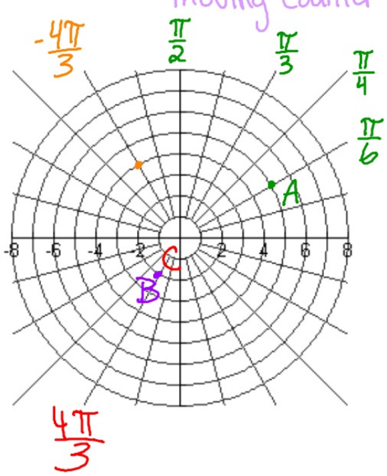
Polar Functions

(r, θ)

r = ______

↑ radius (distance from origin)

↖ angle (from standard position) (starting on the positive x-axis + moving counter-clockwise)



Plot:

A - $(5, \frac{\pi}{6})$

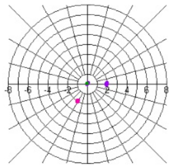
B - $(-2, \frac{\pi}{3})$

C - $(2, \frac{4\pi}{3})$

D - $(4, -\frac{4\pi}{3})$

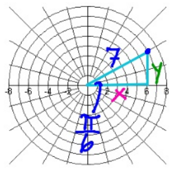
$r = 2\cos(3\theta)$

θ	r
0	2
$\frac{\pi}{6}$	0
$\frac{\pi}{3}$	-2
$\frac{\pi}{2}$	0



A: $(7, \frac{\pi}{6})$

find the x + y-coordinates



$\sin(\frac{\pi}{6}) = \frac{y}{7} \rightarrow y = 7\sin(\frac{\pi}{6})$

$\cos(\frac{\pi}{6}) = \frac{x}{7} \rightarrow x = 7\cos(\frac{\pi}{6})$

To convert to x + y coordinates

$y = r\sin\theta$

$x = r\cos\theta$

What does $\frac{dr}{d\theta}$ mean?

derivative of r in terms of θ → rate that the radius is changing with respect to the angle

What does the slope of a line represent?

$\frac{\text{change in } y}{\text{change in } x}$

How can we find $\frac{dy}{dx}$ with polar functions?

① convert to x + y

$x = r\cos\theta$

$y = r\sin\theta$

② find $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$

③ divide

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

find the equation of the line tangent to $r = \cos 2\theta$ at $\theta = \frac{\pi}{2}$

point

$x = r\cos\theta$

$x = \cos 2\theta \cdot \cos\theta$

$x = \cos\left(2\cdot\frac{\pi}{2}\right)\cdot\cos\left(\frac{\pi}{2}\right)$

$x = 0$

$y = r\sin\theta$

$y = \cos 2\theta \cdot \sin\theta$

$y = \cos\left(2\cdot\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right)$

$y = -1$

$$\text{slope} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-2\sin(2\theta)\cdot\sin\theta + \cos\theta\cdot\cos(2\theta)}{-2\sin(2\theta)\cdot\cos\theta - \sin\theta\cdot\cos(2\theta)}$$

at $\theta = \frac{\pi}{2}$

$$= \frac{-2\sin(\pi)\sin\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{2}\right)\cos(\pi)}{-2\sin(\pi)\cos\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right)\cos(\pi)} = \frac{0}{1} = 0$$

$$y + 1 = 0(x - 0) \longrightarrow y = -1$$