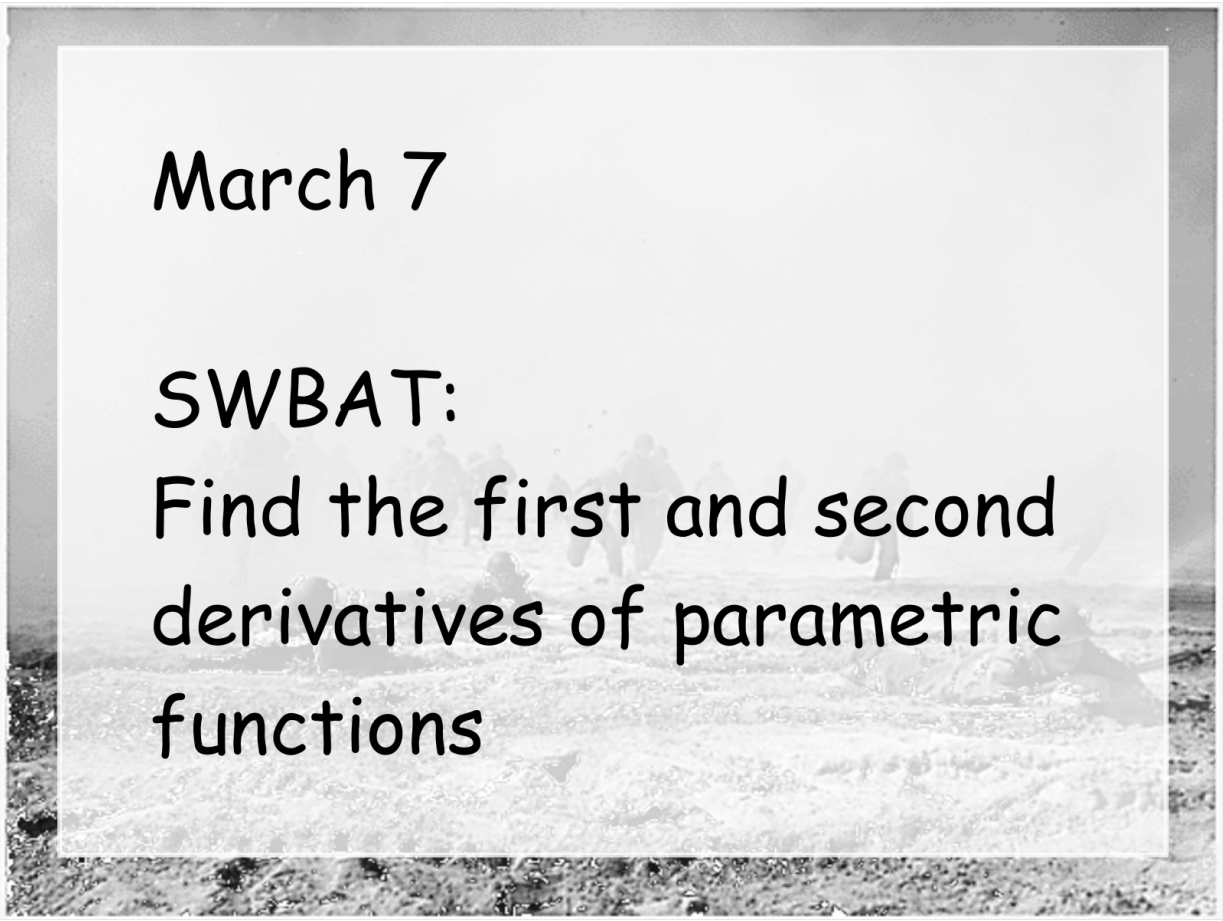

Essential Learning Goals

* Analyze curves in parametric form, polar form and vector form
* Calculate and interpret the derivative of curves given in parametric form, polar form and vector form - including velocity and acceleration
* Calculate the area of a region defined by polar curves.
* Calculate the length of parametric curves.

What does it mean?

$\frac{dy}{dx}$ The instantaneous rate y changes with respect to x

$\frac{dy}{dt}$ The instantaneous rate y changes with respect to t

$\frac{dx}{dt}$ The instantaneous rate x changes with respect to t

If you have $\frac{dy}{dt}$ and $\frac{dx}{dt}$, how can you get $\frac{dy}{dx}$?

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{y'}{x'} \rightarrow \frac{d}{dt}\left(\frac{y'}{x'}\right) \nearrow \frac{y''x' - x''y'}{(x')^2} = \frac{y''}{(x')} - \frac{x''y'}{(x')^2}$$

(equation 7)

derivative of the derivative first

what about $\frac{d^2y}{dx^2}$?

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

(equation 11)

derivative of x

$$= \frac{y'}{x'} \rightarrow \frac{d}{dt}\left(\frac{y'}{x'}\right) \nearrow \frac{y''x' - x''y'}{(x')^2}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right) \cdot \frac{dt}{dx}$$

$$= \frac{d}{dt}\left(\frac{dy}{dx}\right) \cdot \frac{1}{\frac{dx}{dt}}$$

$$= \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

$y(t) = t^2 - t$

$x(t) = t^2 + t$

find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

$$\frac{dy}{dt} = 2t - 1$$

$$\frac{dx}{dt} = 2t + 1$$

$$\frac{dy}{dx} = \frac{2t-1}{2t+1}$$

$$\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{2(2t+1) - (2t-1)2}{(2t+1)^2}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{2(2t+1) - (2t-1)2}{(2t+1)^2}}{(2t+1)} = \frac{2(2t+1) - 2(2t-1)}{(2t+1)^3}$$

$x(t) = \ln(5t)$

$y(t) = e^{5t}$

find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

$\frac{dy}{dt}$ $\quad y'(t) = 5e^{5t}$

$\frac{dx}{dt}$ $\quad x'(t) = \frac{1}{5t}\left(\frac{5}{t}\right)$

$\frac{dy}{dx}$ $\quad \frac{dy}{dx} = \frac{5e^{5t}}{\frac{1}{t}} = 5te^{5t}$

$\frac{d}{dt}\left(\frac{dy}{dx}\right)$ $\quad \frac{d^2y}{dt^2} = 25te^{5t} + 5e^{5t}$

$\frac{d^2y}{dx^2}$ $\quad \frac{d^2y}{dx^2} = \frac{25te^{5t} + 5e^{5t}}{\frac{1}{t}} = t\left(25te^{5t} + 5e^{5t}\right)$

$25t^2e^{5t} + 5te^{5t}$