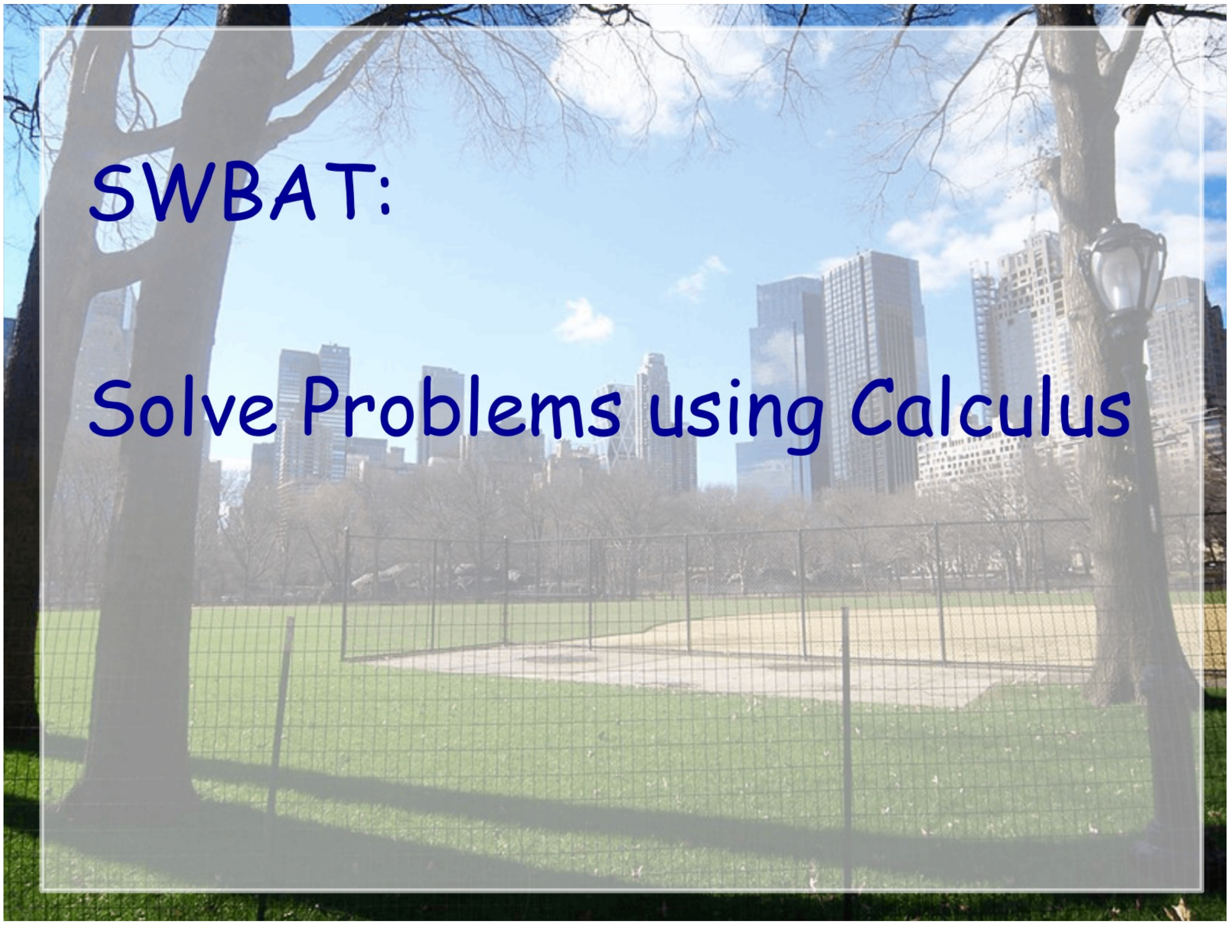


Curve Analysis

The first derivative $\left(\frac{dy}{dx}\right)$ gives me information about the slope of the function.

- The function is increasing when the derivative is positive.
- The function is decreasing when the derivative is negative.
- When the derivative is zero or undefined, the function has a possible maximum or minimum.

The second derivative $\left(\frac{d^2y}{dx^2}\right)$ gives me information about the concavity of the function.

- The function is concave up when the second derivative is positive.
- The function is concave down when the second derivative is negative.
- When the second derivative is zero or undefined, the function has a possible inflection point.

To find maximums and minimums:

- Critical Points are where the first derivative equals zero or is undefined.

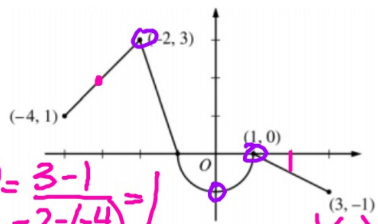
First Derivative Test:

Using a sign chart...

- If the first derivative changes from positive to negative, the function has a maximum.
- If the first derivative changes from negative to positive, the function has a minimum.

Second Derivative Test:

- If the value of the second derivative at the critical point is positive, the function is concave up and has a minimum.
- If the value of the second derivative at the critical point is negative, the function is concave down and has a maximum.



$g''(-3) = f'(-3) = \frac{3-1}{-2-(-4)} = 1$

Graph of f $= g'(x)$

$g(2) = \int_1^2 f(t)\,dt = \frac{-1}{4}$

$g(-2) = \int_1^{-2} g(t)\,dt = -\int_{-2}^{1} g(t)\,dt$

$= \frac{\pi}{2} - \frac{3}{2}$

3. Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.

(a) Find the values of $g(2)$ and $g(-2)$.

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

-2

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

$x = -1, 1$

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

$g''(x) = 0$ or undefined + change sign

$g''(x) = f'(x) =$ slope of f

x	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

4. The function f is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of f', the derivative of f, are given for selected values of x in the table above.

(a) Write an equation for the line tangent to the graph of f at $x = 1$. Use this line to approximate $f(1.4)$.

(b) Use a midpoint Riemann sum with two subintervals of equal length and values from the table to approximate $\int_1^{1.4} f'(x)\,dx$. Use the approximation for $\int_1^{1.4} f'(x)\,dx$ to estimate the value of $f(1.4)$. Show the computations that lead to your answer.

(c) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the computations that lead to your answer. $f'(x) = \frac{dy}{dx} \rightarrow dy = f'(x)dx$

(d) Write the second-degree Taylor polynomial for f about $x = 1$. Use the Taylor polynomial to approximate $f(1.4)$.

$\int_1^{1.4} f'(x)\,dx \approx .2(10) + .2(13)$

$\int_1^{1.4} f'(x)\,dx = f(1.4) - f(1)$

$.2(23) = f(1.4) - 15$

$.2(23) + 15 = f(1.4)$

$y - 15 = 8(x-1)$ $\quad$ $y - 15 = 8(1.4-1)$

$y = 8(1.4-1) + 15$

Old x	Old y	dx	$dy = f'(x)dx$	New x	New y
1	15	.2	$f'(1)(.2) = 8(.2) = \frac{8}{5} = 1.6$	$1.2 = 1 + .2$	$15 + \frac{8}{5} = 16.6$
1.2	16.6	.2	$f'(1.2)(.2) = 12(.2) = 2.4$	1.4	$16.6 + 2.4 = 19$

$f(1.4) \approx 19$

$f(1) + f'(1)(x-1) + \frac{f''(1)(x-1)^2}{2!}$

$15 + 8(x-1) + \frac{20(x-1)^2}{2!}$

$15 + 8(x-1) + 10(x-1)^2$

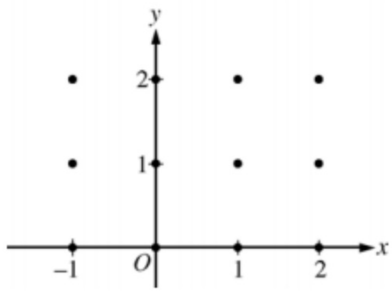
$f(1.4) \approx 15 + 8(1.4-1) + 10(1.4-1)^2$

Y/N

4. Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and sketch the solution curve that passes through the point $(0, 1)$.

(Note: Use the axes provided in the pink test booklet.)



(b) The solution curve that passes through the point $(0, 1)$ has a local minimum at $x = \ln\left(\frac{3}{2}\right)$. What is the y-coordinate of this local minimum?

(c) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(-0.4)$. Show the work that leads to your answer.

(d) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine whether the approximation found in part (c) is less than or greater than $f(-0.4)$. Explain your reasoning.