Essential Learning Goals:

- Find specific antiderivatives using inital conditions
- Solve Separable differential equations and use them in modeling - including exponential and logistic growth problems
- Find the length of a curve over a given interval

November 15

How is Euler's method related to Reimann Sums (RAM)?

November 15

Students will verbally explain how to find the solution to a differential equation

(using the words:
separate, initial value, constant ...)

$\frac{dy}{dx} = \frac{1}{x^2} + 4x^3$

find the general solution for the above differential equation

$(y = \_\_\_\_)$

① separate variables

$dx \cdot \frac{dy}{dx} = \frac{1}{x^2} + 4x^3 \, dx$

$dy = \left(\frac{1}{x^2} + 4x^3\right) dx$

② integrate both sides

$\int dy = \int \frac{1}{x^2} + 4x^3 \, dx$

$y = \int x^{-2} + 4x^3 \, dx$

$y = \frac{x^{-1}}{-1} + x^4 + C$

③ solve for y

$y = -\frac{1}{x} + x^4 + C$

Solve the separable differential equation $\frac{dy}{dx} = 3x^2 y$ if $y = 5$ when $x = 0$

① separate

$dx \quad \frac{dy}{dx} = 3x^2 y \quad dx$

$\frac{dy}{y} = \frac{3x^2 y \, dx}{y}$

$\frac{dy}{y} = 3x^2 \, dx$

② integrate

$\int \frac{dy}{y} = \int 3x^2 \, dx$

$\int \frac{1}{y} \, dy = \int 3x^2 \, dx$

$\ln|y| = x^3 + C$

③ solve for y

$e^{\ln|y|} = e^{x^3 + C}$

$|y| = e^{x^3 + C} = e^{x^3}(e^C) = Ce^{x^3}$

④ use initial conditions

$x = 0, \; y = 5$

$|y| = Ce^{x^3}$

$|5| = Ce^{0^3}$

$5 = Ce^0 = C(1) = C$

⑤ plug C into equation

$|y| = 5e^{x^3}$

⑥ Check i.c.

$y = 5e^{x^3}$

$5 = 5e^0$ ✓

---

find the particular solution to the separable differential equation $\frac{dy}{dx} = \frac{x+1}{y}$ with initial condition $y(2) = -4$

$y = \sqrt{x^2 + 2x + C}$

$y = -\sqrt{x^2 + 2x + 8}$

↑ negative because initial condition is -4

$(x = 2, \; y = -4)$