November 16

SWBAT:

Determine if an intergral converges or diverges.

Improper Integrals

Integral with undefined/infinite limits of integration (bounds)

$$\int_a^{\infty} f(x)\,dx = \lim_{R\to\infty} \int_a^R f(x)\,dx$$

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{R\to-\infty} \int_R^b f(x)\,dx$$

$$\int_1^3 \frac{1}{\sqrt{3-x}}\,dx = \lim_{R\to 3^-} \int_1^R \frac{1}{\sqrt{3-x}}\,dx$$

Converge

If the limit exists the integral converges to the value of the limit

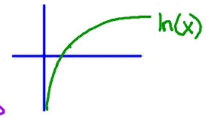
Diverge

If the limit does not exist (DNE or $\pm\infty$) the integral diverges

$$\int_1^\infty \frac{1}{x}\,dx$$

$$\lim_{R\to\infty} \int_1^R \frac{1}{x}\,dx$$

$$= \lim_{R\to\infty} \ln(x)\Big|_1^R = \lim_{R\to\infty} \ln(R) - \ln(1) = \infty$$



$\int_1^\infty \frac{1}{x}\,dx$ diverges

$$\int_1^\infty \frac{1}{x^2}\,dx$$

$$\lim_{R\to\infty} \int_1^R \frac{1}{x^2}\,dx = \lim_{R\to\infty} \int_1^R x^{-2}\,dx$$

$$= \lim_{R\to\infty} \frac{x^{-1}}{-1}\Big|_1^R = \lim_{R\to\infty} -\frac{1}{x}\Big|_1^R = \lim_{R\to\infty} \frac{-1}{R} + \frac{1}{1} = 0 + 1 = 1$$

$\int_1^\infty \frac{1}{x^2}\,dx$ converges to 1