November 18

What are the important steps when solving a separable differential equation?

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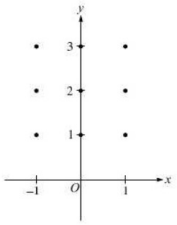
Students will verbally explain how to draw a slope field, apply Euler's method and find the solution to a differential equation

(using the words: separate, initial value, constant ...)

1998 Calculus BC Free-Response Questions

4. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

   (a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.



   (b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 3$. Use Euler's method starting at $x = 0$, with a step size of 0.1, to approximate $f(0.2)$. Show the work that leads to your answer.

   (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$. Use your solution to find $f(0.2)$.

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1998 Calculus BC Scoring Guidelines

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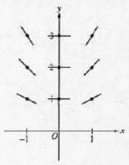
(a)



1: line segments at nine points with negative - zero - positive slope left to right and increasing steepness bottom to top at $x = 1$ and $x = -1$

(b) $f(0.1) \approx f(0) + f'(0)(0.1)$

$= 3 + \frac{1}{2}(0)(3)(0.1) = 3$

$f(0.2) \approx f(0.1) + f'(0.1)(0.1)$

$= 3 + \frac{1}{2}(0.1)(3)(0.1)$

$= 3 + \frac{.03}{2} = 3.015$

2: 1: Euler's Method equations or equivalent table; 1: answer (not eligible without first point)

Special Case: 1/2 for first iteration 3.015 and second iteration 3.045

O_x	O_y	dx	dy	N_x	N_y
0	3	.1	0	.1	3
.1	3	.1	.015	.2	3.015

(c) $\frac{dy}{dx} = \frac{xy}{2}$

$\int \frac{dy}{y} = \int \frac{x}{2}\,dx$

$\ln|y| = \frac{1}{4}x^2 + C_1$

$y = Ce^{x^2/4}$

$3 = Ce^0 \implies C = 3$

$y = 3e^{x^2/4}$

$f(0.2) = 3e^{.04/4} = 3e^{.01} = 3.030$

6: 1: separates variables; 1: antiderivative of dy term; 1: antiderivative of dx term; 1: solves for y; 1: solves for constant of integration; 1: evaluates $f(0.2)$

Note: max 4/6 [1-1-1-0-0-1] if no constant of integration