November 26

SWBAT:
determine the convergence of an improper integral using the comparison test

COMPLETE EACH COMPARISON WITH

$\leq$ OR $\geq$

$$\frac{1}{1+n^2} \leq \frac{1}{n^2}$$

$$\frac{1}{2+\sqrt{n}} \leq \frac{1}{\sqrt{n}}$$

$$\int_1^\infty \frac{1}{\sqrt{1+n^4}}\,dn \leq \int_1^\infty \frac{1}{n^2}\,dn$$

$$\int_3^\infty \frac{\ln z}{z}\,dz \geq \int_3^\infty \frac{1}{z}\,dz$$

$z > e$

$$\frac{1}{2+\sqrt{n}} \geq \frac{1}{n}$$

$n > 4$

Comparison Test

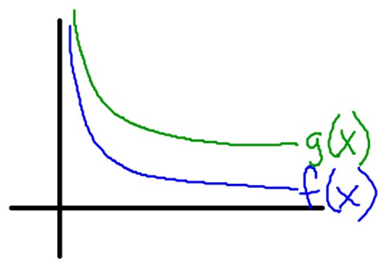
f and g are continuous functions on $[a, \infty)$, where $a > 0$ and $0 \leq f \leq g$

$\int_a^\infty f(x)\,dx$ converges if $\int_a^\infty g(x)\,dx$ converges

$\int_a^\infty g(x)\,dx$ diverges if $\int_a^\infty f(x)\,dx$ diverges

If the bigger function converges then the smaller function converges

If the smaller function diverges then the bigger function diverges



Determine if the integral converges or diverges

$\int_1^\infty \frac{1}{x^3+1}\,dx$

$$0 \leq \frac{1}{x^3+1} \leq \frac{1}{x^3}$$

$\int_1^\infty \frac{1}{x^3}\,dx$ converges because $p=3>1$

by the comparison test $\int_1^\infty \frac{1}{x^3+1}\,dx$ converges

$\int_3^\infty \frac{\ln x}{x}\,dx$

$$0 \leq \frac{\ln x}{x} \qquad \frac{1}{x} \leq \frac{\ln x}{x}$$

$\int_3^\infty \frac{1}{x}\,dx$ diverges because $p=1$

by the comparison test $\int_3^\infty \frac{\ln x}{x}\,dx$ diverges

$$\int_1^\infty \frac{1}{3+\sqrt{x}}\,dx$$

$$0 \le \frac{1}{3+\sqrt{x}}$$

$$\frac{1}{3+\sqrt{x}} \le \frac{1}{\sqrt{x}}$$

$\int_1^\infty \frac{1}{\sqrt{x}}\,dx$ diverges because $p = \frac{1}{2} \le 1$

$$\frac{1}{3+\sqrt{x}} \ge \frac{1}{x} \quad \text{if } x \ge 9$$

$$\int_1^\infty \frac{1}{3+\sqrt{x}}\,dx = \underbrace{\int_1^9 \frac{1}{3+\sqrt{x}}\,dx}_{\text{converge}} + \int_9^\infty \frac{1}{3+\sqrt{x}}\,dx$$

$\frac{1}{3+\sqrt{x}} \ge \frac{1}{x}$ + $\int_9^\infty \frac{1}{x}\,dx$ diverges

by comparison test $\int_9^\infty \frac{1}{3+\sqrt{x}}\,dx$ diverges

$$\boxed{\int_1^\infty \frac{1}{3+\sqrt{x}}\,dx \text{ diverges}}$$