November 27

SWBAT:
Find the general and explicit solution for a differential equation

Essential Learning Goals:

- Find specific antiderivatives using inital conditions
- Solve Separable differential equations and use them in modeling - including exponential and logistic growth problems
- Find the length of a curve over a given interval

$\frac{dy}{dx} = \frac{1}{x^2} + 4x^3$

find the general solution ($y =$ ____)

$\int (\quad) dx$

① separate variables

$dx \cdot \frac{dy}{dx} = \frac{1}{x^2} + 4x^3 \cdot dx$

$dy = \frac{1}{x^2} + 4x^3\, dx$

② integrate both sides

$\int 1\, dy = \int \frac{1}{x^2} + 4x^3\, dx$

$y + C_1 = -x^{-1} + \frac{4x^4}{4} + C_2$

$y = -x^{-1} + x^4 + C$

③ solve for y

$y = -\frac{1}{x} + x^4 + C$

find the general solution

$y' = 3x^2 y$

$\frac{dy}{dx} = 3x^2 y$

① separate variables

$\frac{dx}{y} \cdot \frac{dy}{dx} = \frac{3x^2 y}{y} \cdot dx$

$\frac{1}{y} dy = 3x^2 dx$

② integrate both sides

$\int \frac{1}{y} dy = \int 3x^2 dx$

$\ln|y| = x^3 + C$

③ solve for y

$\ln|y| = x^3 + C$

$e \qquad e$

$|y| = e^{x^3 + C}$ $\quad (a^x \cdot a^y = a^{x+y})$

$|y| = e^{x^3} \cdot e^C$

$|y| = Ce^{x^3}$

$y = \pm Ce^{x^3} \rightarrow y = Ce^{x^3}$

Solve the initial value problem if $y' = \frac{x+1}{y}$ and $y(2) = -4$

$$\frac{dy}{dx} = \frac{x+1}{y}$$

① separate

$$y \cdot dy = (x+1)\,dx$$

②③ integrate

$$\int y\,dy = \int x+1\,dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + x + C$$

③ solve for y

$$y^2 = x^2 + 2x + C$$

$$y = \sqrt{x^2+2x+C} \quad \text{or} \quad -\sqrt{x^2+2x+C}$$

④ use initial condition $y(2) = -4$

$$y = \pm\sqrt{x^2+2x+C}$$

$$-4 = \pm\sqrt{2^2+2(2)+C}$$

⑤ solve for C

$$16 = 4+4+C$$

$$8 = C$$

⑥ plug C into your final equation

$$y = -\sqrt{x^2+2x+8}$$

↑ negative because I.C. is $y = -4$