November 29

SWBAT:
Solve Separable Differential Equations

A hot anvil with cooling constant $k = 0.02\text{s}^{-1}$ is submerged in a large pool of water whose temperature is 10°C. Let $y(t)$ be the anvil's temperature t seconds later.

What is the differential equation satisfied by $y(t)$?

Use Newton's Law of Cooling:
$y' = -k(y - T_0)$

$\frac{dy}{dt}$ cooling constant temp outside

$$y' = -.02(y-10)$$

Find a formula for $y(t)$, assuming the object's intial temperature is 100°C.

$$y(0) = 100$$

$$\frac{dy}{dt} = -.02(y-10)$$

$$\frac{dy}{y-10} = -.02\,dt$$

$$\int \frac{dy}{y-10} = \int -.02\,dt$$

$$\ln|y-10| = -.02t + C$$

$$\ln|100-10| = -.02(0) + C$$

$$\ln 90 = C$$

$$\ln|y-10| = -.02t + \ln 90$$

e e

$$|y-10| = e^{-.02t+\ln 90}$$

$$|y-10| = 90e^{-.02t}$$

$$y = 90e^{-.02t} + 10$$

How long does it take the object to cool down to 20°C?

$$y = 90e^{-.02t} + 10$$

$$20 = 90e^{-.02t} \cancel{+10}$$

$$10 = 90e^{-.02t}$$

$$\frac{10}{90} = e^{-.02t}$$

$$\ln(1/9) = \ln(e^{-.02t})$$

$$\ln(1/9) = -.02t$$

$$t = \frac{\ln(1/9)}{-.02} = 109.861 \text{ sec}$$

A cup of soup with cooling constant k = 0.012 sec^{-1} is placed in a rooom at temperature 25°C.

How fast is the soup cooling (in degrees per second) when its temperature is T = 75°C?

$y' = -k(y - T_0)$

$y' = -.012(y-25)$

$y' = -.012(75-25)$

$y' = -.6°/\text{sec}$

Use a linear approximation to estimate the change in temperature over the next 5 seconds when T = 75°C.

$y - y_1 = m(x - x_1)$

initial point $(0,0)$

slope $= y' = -.6$

$y - 0 = -.6(x - 0)$

$y = -.6(5) = -3\ °C$

$y' = -.6°/\text{sec}$

5 sec

If the soup is served at 80°C, how long will it take to reach an optimal temperature of 60°C?

$y(0) = 80$

$y(t) = ?$

$\frac{dy}{dt} = -.012(y-25)$

$y = 55e^{-.012t} + 25$

$t = 37.665$