November 5

Why are integrals needed to find the area, volume and average value?

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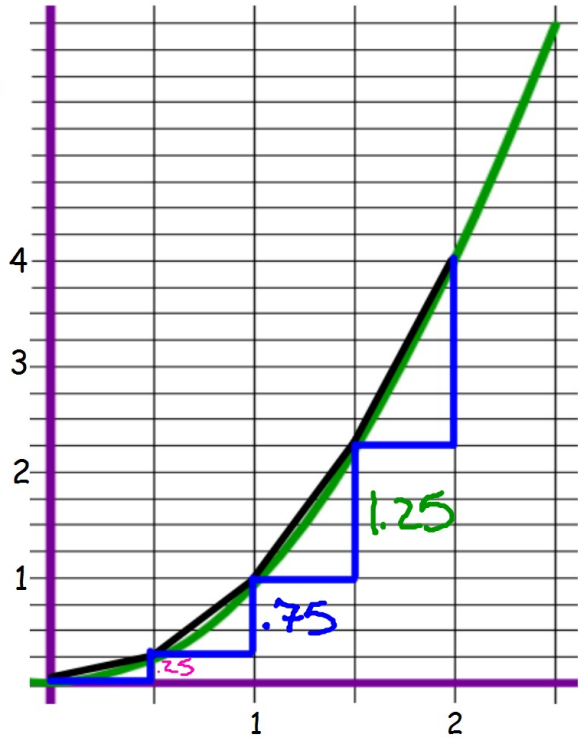
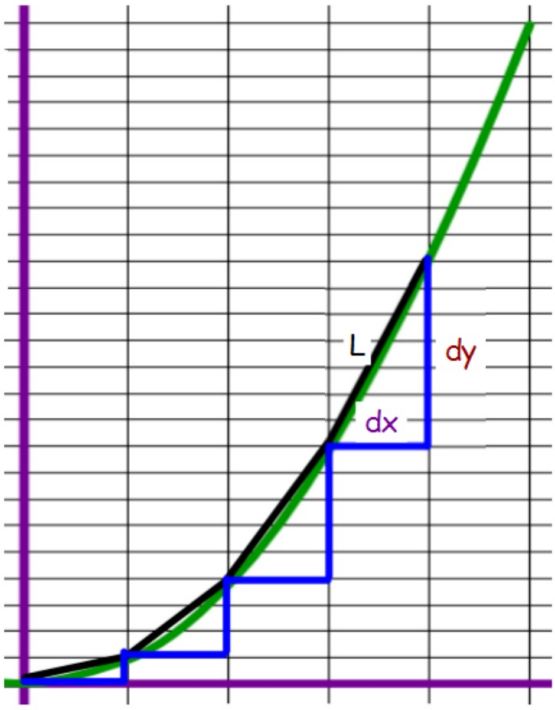
Students will verbally explain how to find the length of a curve

(using the words:
integral, interval, derivative...)

Find the length of $y = x^2$ on $[0, 2]$



$$\sqrt{.5^2 + .25^2} + \sqrt{.5^2 + .75^2} + \sqrt{.5^2 + 1.25^2} + \sqrt{.5^2 + 1.75^2} \approx 4.627$$



$$L = \sqrt{(dx)^2 + (dy)^2}$$

$$L = \sqrt{(dx)^2 + (dy)^2}\ \frac{(dx)}{(dx)}$$

$$L = \sqrt{\frac{(dx)^2}{(dx)^2} + \frac{(dy)^2}{(dx)^2}}\ dx$$

$$L = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

Arc Length

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

Find the length of $y = \tan x$ on $[-\pi/3, 0]$

$y = \tan x$

$\frac{dy}{dx} = \sec^2 x$

$$L = \int_{-\frac{\pi}{3}}^{0} \sqrt{1 + (\sec^2 x)^2}\, dx = \int_{-\frac{\pi}{3}}^{0} \sqrt{1 + \sec^4 x}\, dx$$

$= 2.056$

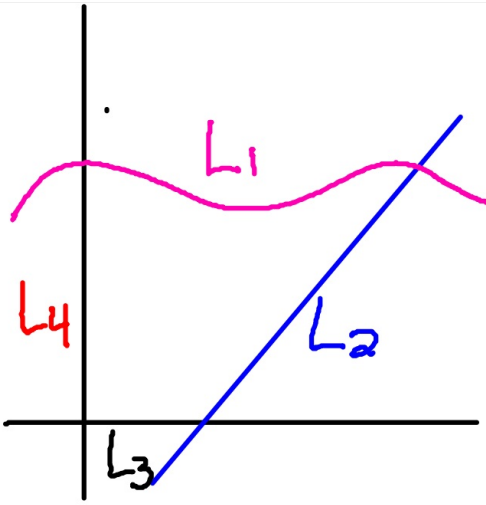
Find the length of $x = \sqrt{1 - y^2}$ on $[-1/2, 1/2]$

y-values

$\frac{dx}{dy} = \frac{1}{2}(1 - y^2)^{-1/2}(-2y)$

$= -y(1 - y^2)^{-1/2}$

$$L = \int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt{1 + \left(-y(1-y^2)^{-1/2}\right)^2}\, dy = 1.047$$



$y = \cos x + 3$

$y = -2 + \frac{x}{3}$

Find the Perimeter of lake in Quadrant I

$$\int_0^A \sqrt{1+(-\sin x)^2}\,dx + \int_6^A \sqrt{1+\left(\frac{1}{3}\right)^2}\,dx$$

$$+6 + 4 = 36.303$$