October 16

Why is the u-substitution technique helpful? When is it necessary?

October 16

Students will verbally explain how to find the area bounded by two functions

(using the words:
above, below, right, left...)

Find the area bounded by

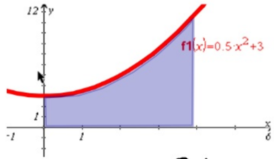
$f(x) = 0.5x^2 + 3$

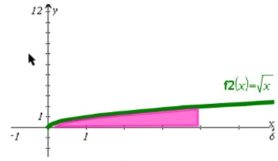
$g(x) = \sqrt{x}$

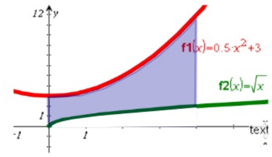
$x = 4$

and the y-axis

$(x=0)$







$$\int_0^4 f(x)\,dx - \int_0^4 g(x)\,dx$$

$$\int_0^4 f(x) - g(x)\,dx$$

$$\int_0^4 \frac{1}{2}x^2 + 3 - (\sqrt{x})\,dx$$

$$= \int_0^4 \frac{1}{2}x^2 + 3 - x^{1/2}\,dx = \frac{1}{2}\left(\frac{x^3}{3}\right) + 3x - \frac{x^{3/2}}{\frac{3}{2}}\Bigg|_0^4$$

$$\frac{1}{2}\int x^2\,dx$$

$$= \frac{x^3}{6} + 3x - \frac{2x^{3/2}}{3}\Bigg|_0^4$$

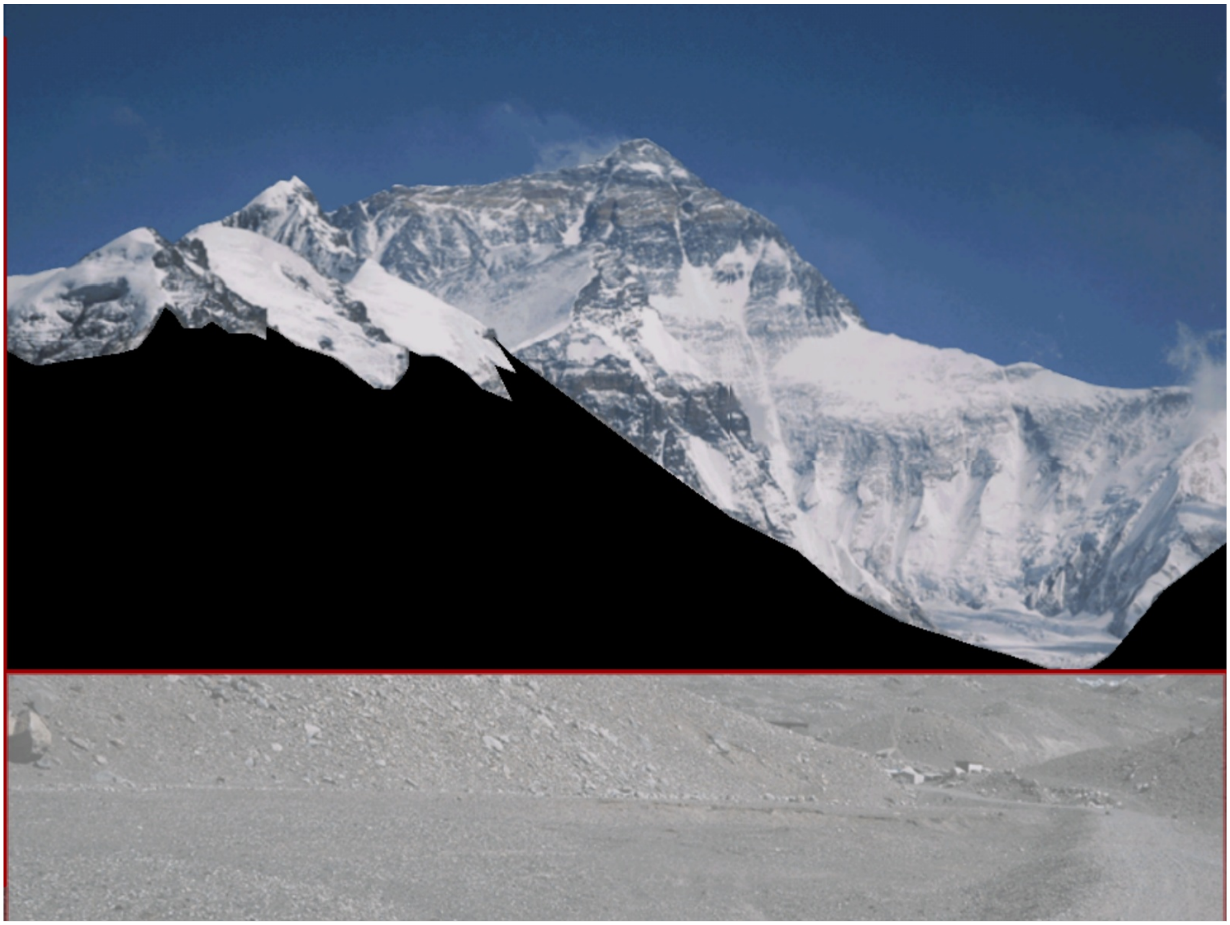
$$= \frac{4^3}{\ } + 3(4) - \frac{2(\sqrt{4})^3}{\ } \quad (0)$$

Area between two curves

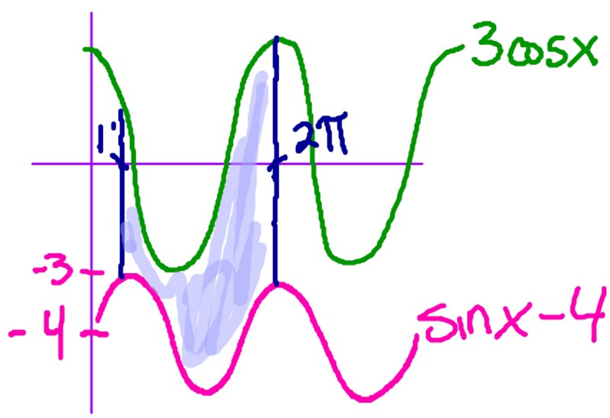
$$\int_a^b f(x) - g(x)\,dx$$

where $f(x) \geq g(x)$

$$\int_a^b \underset{y}{\text{upper}} - \underset{y}{\text{lower}}\,dx$$



Find the area bounded by
$f(x) = 3\cos x$
$g(x) = \sin x - 4$
on the interval
$(1, 2\pi)$



$$\int_1^{2\pi} 3\cos x - (\sin x - 4)\,dx$$

$$= 3\sin x - (-\cos x - 4x)\Big|_1^{2\pi}$$

$$= 3\sin(2\pi) + \cos(2\pi) + 4(2\pi) - (3\sin(1) + \cos(1) + 4)$$

$$= 1 + 8\pi - 3\sin(1) - \cos(1) - 4$$

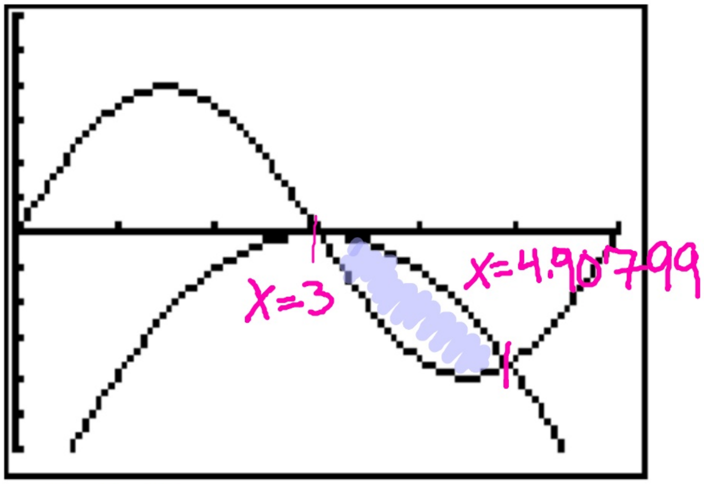
$$= 8\pi - 3\sin(1) - \cos(1) - 3$$

Find the area enclosed by

$y = 4\sin\left(\frac{\pi x}{3}\right)$

and

$y = -(x - 3)^2$



$$\int_{3}^{4.907} -(x-3)^2 - 4\sin\left(\frac{\pi x}{3}\right) dx$$

$$= 3.087$$