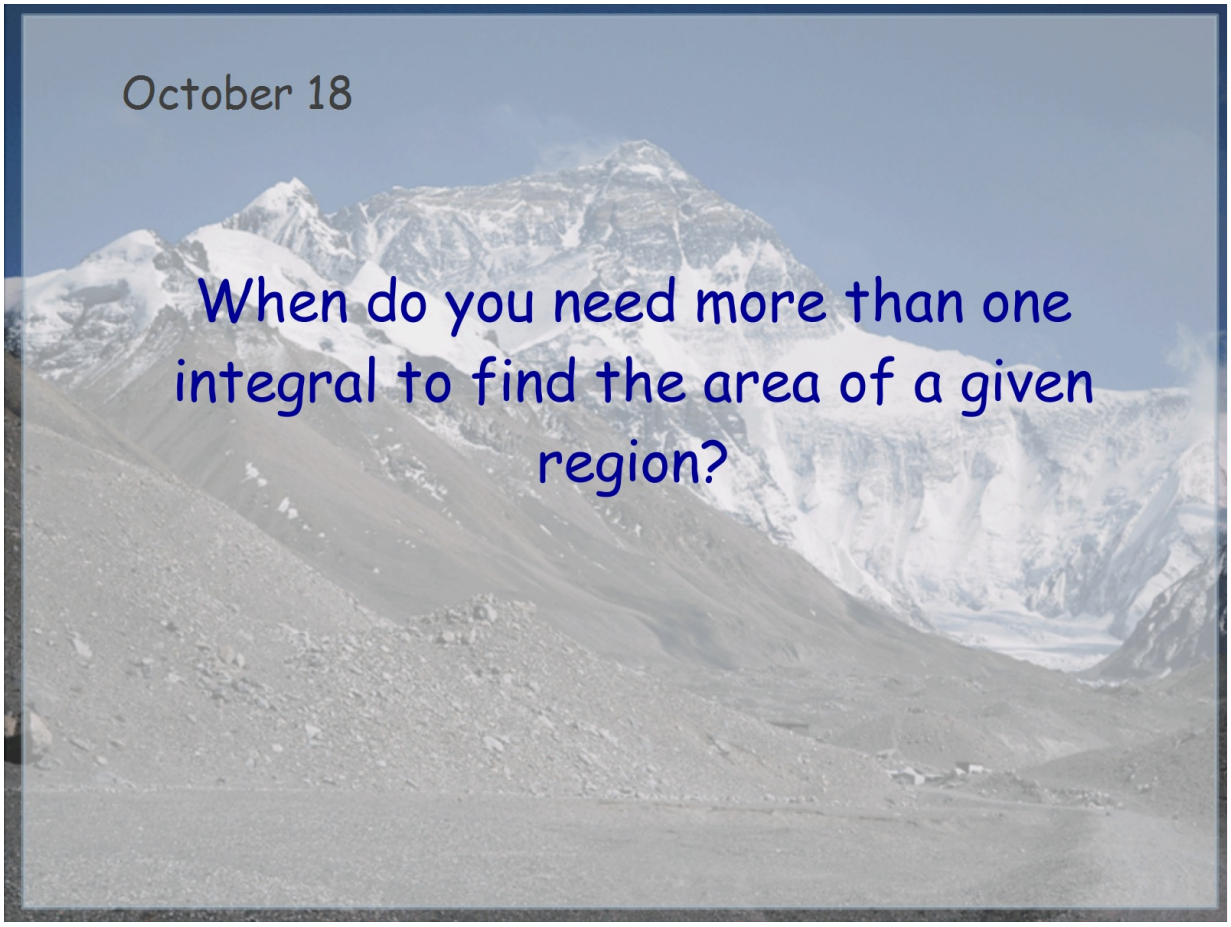
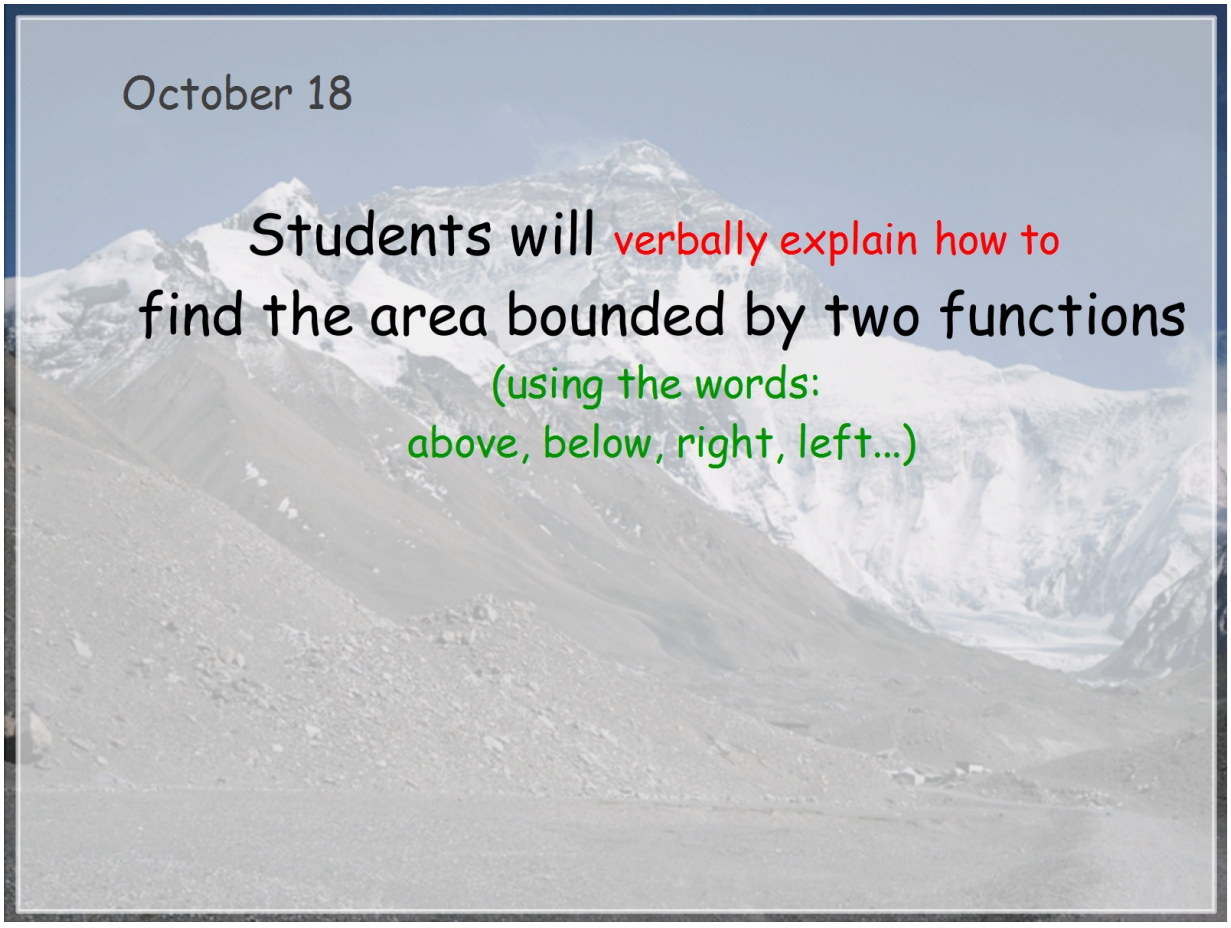
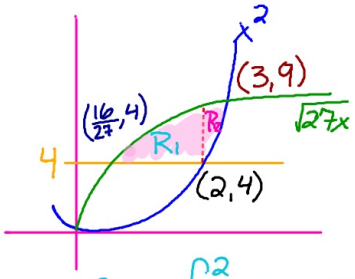

October 18
When do you need more than one integral to find the area of a given region?


October 18
Students will verbally explain how to find the area bounded by two functions
(using the words:
above, below, right, left…)

Find the area enclosed by $y = x^2$, $y = \sqrt{27x}$ and below by $y = 4$



$x^2 = 4$
$x = 2$

$x^2 = \sqrt{27x}$
$x^2 = \sqrt{27}\sqrt{x}$
$x^4 = 27x$
$-27x \quad -27x$
$x^4 - 27x = 0$
$x(x^3 - 27) = 0$
$x = 0 \quad x^3 - 27 = 0$
$x^3 = 27$
$x = \sqrt[3]{27}$
$x = 3$

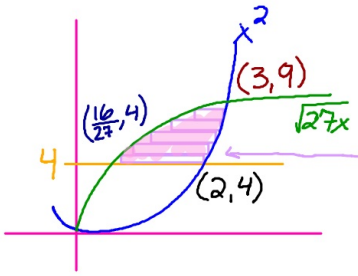
$\sqrt{27x} = 4$
$27x = 16$
$x = \frac{16}{27}$

$$A_{R_1} = \int_{\frac{16}{27}}^{2} \sqrt{27x} - 4 \, dx = 2.588$$

$$A_{R_2} = \int_{2}^{3} \sqrt{27x} - x^2 \, dx = 1.868$$

Total Area = 4.456

Find the area enclosed by $y = x^2$, $y = \sqrt{27x}$ and below by $y = 4$



width is change in y = dy

length is right function − left function

- put equations in terms of y ($x = \_\_\_$)

$y = x^2 \longrightarrow x = \sqrt{y}$

$y = \sqrt{27x} \longrightarrow x = \frac{y^2}{27}$

- Area

$$\int_4^9 \sqrt{y} - \frac{y^2}{27} \, dy = 4.456$$

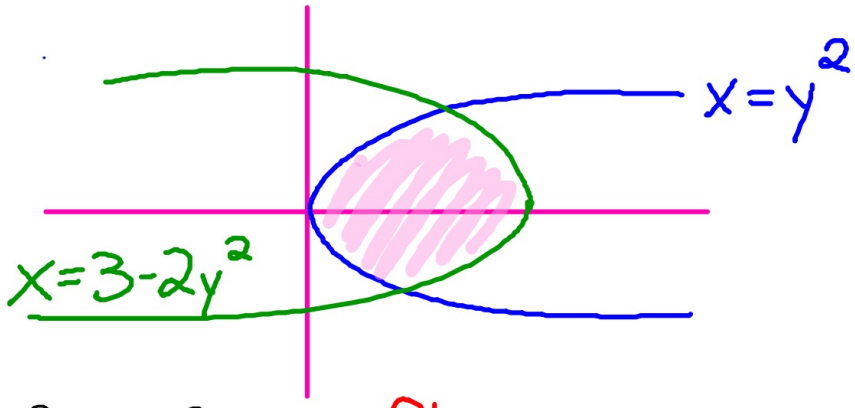
Area between curves with respect to y

$$\int_c^d \text{right}_x - \text{left}_x \, dy$$

↑ y-values

* equations should be $x =$ ____

Find the area enclosed by $x = y^2$ and $x = 3 - 2y^2$



$y^2 = 3 - 2y^2$

$\frac{0}{3} = \frac{3}{3} - \frac{3y^2}{3}$

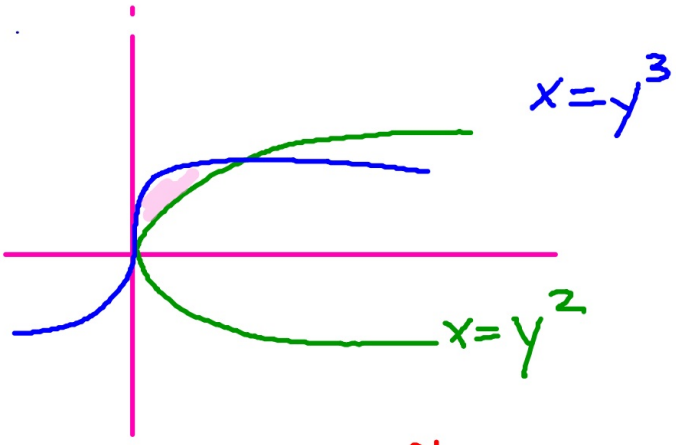
$0 = 1 - y^2$

$1 = y^2 \rightarrow y = \pm 1$

$$A = \int_{-1}^{1} 3 - 2y^2 - y^2 \, dy$$

$= 4$

Find the area enclosed by $y^3 - x = 0$ and $x - y^2 = 0$



$y^3 = y^2$

$y^3 - y^2 = 0$

$y^2(y-1) = 0$

$y = 0, y = 1$

$$A = \int_0^1 y^2 - y^3 \, dy = \frac{1}{12}$$

$$A = \int_0^1 \sqrt{x} - \sqrt[3]{x} \, dx = \frac{1}{12}$$