October 2

How are the antiderivative, function and derivative all related?

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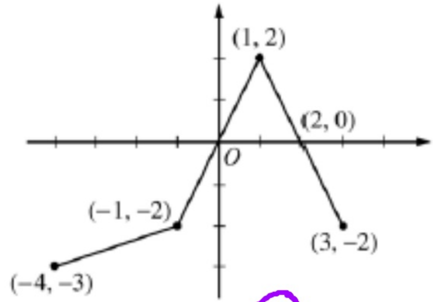
Students will verbally explain how to apply the fundamental theorem of calculus to graphical problems

(using the words:
integral, area, slope, derivative, zero...)



Graph of f

straight lines

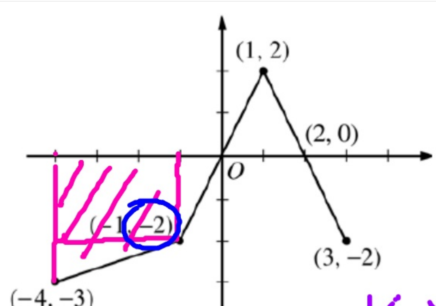
4. The graph of the function f above consists of three line segments.

(a) Let g be the function given by $g(x) = \int_{-4}^{x} f(t)\,dt$. For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

(b) For the function g defined in part (a), find the x-coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.

(c) Let h be the function given by $h(x) = \int_{x}^{3} f(t)\,dt$. Find all values of x in the closed interval $-4 \le x \le 3$ for which $h(x) = 0$.

(d) For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.



Graph of $f = g'(x)$

$g''(x) = f'(x)$
(slope)

4. The graph of the function f above consists of three line segments.

area value slope

(a) Let g be the function given by $g(x) = \underbrace{\int_{-4}^{x}}$ $f(t)\,dt$. For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

g(x) is the antiderivative of f(x)

$g'(x) = f(x)$

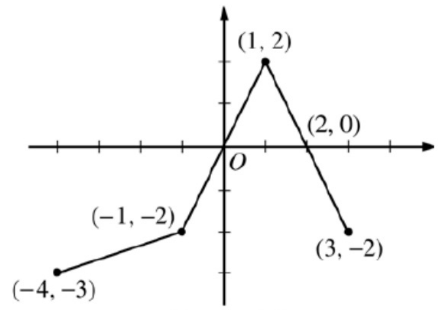
$$g(-1) = \int_{-4}^{-1} f(t)\,dt = -\left[2(3) + \tfrac{1}{2}(1)(3)\right] = -7.5$$

(area from $x = -4$ to $x = -1$)

$$g'(-1) = f(-1) = -2$$

$$g''(-1) = f'(-1) = \text{DNE}$$

$g'(x)=f(x)$



Graph of f

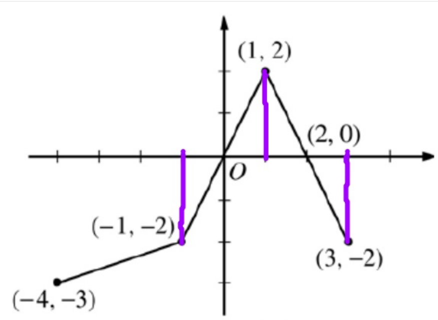
$g''(x)=0$ or DNE & changes sign

4. The graph of the function f above consists of three line segments.

(b) For the function g defined in part (a), find the x-coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.

where does the slope of f change signs?

$x=1$ because the slope of $f(x)$ is undefined and changes from positive to negative



Graph of f

$h'(x)=\frac{d}{dx}\left(\int_x^3 f(t)\,dt\right)$
$=f(3)\cdot 0-f(x)\cdot 1=-f(x)$

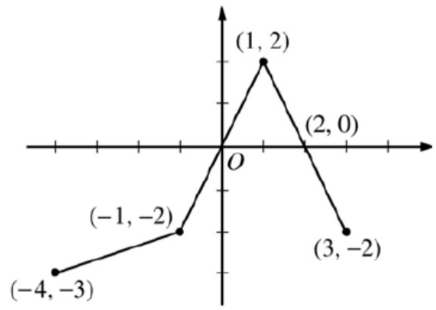
4. The graph of the function f above consists of three line segments.

(c) Let h be the function given by $h(x)=\int_x^3 f(t)\,dt$. Find all values of x in the closed interval $-4 \le x \le 3$ for which $h(x)=0$.

$h'(x)=-f(x)$

$\int_x^3 f(t)\,dt=0 \qquad x=1,-1,3$

Area $=0$



Graph of f

4. The graph of the function f above consists of three line segments.

(c) Let h be the function given by $\underbrace{h(x) = \int_x^3 f(t)\,dt}_{h'(x) = -f(x)}$. Find all values of x in the closed interval $-4 \le x \le 3$ for which $h(x) = 0$.

(d) For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.

$h'(x)$ is negative

$(0, 2)$ because $h'(x) = -f(x)$ and $f(x)$ is postive on $(0, 2)$