October 30

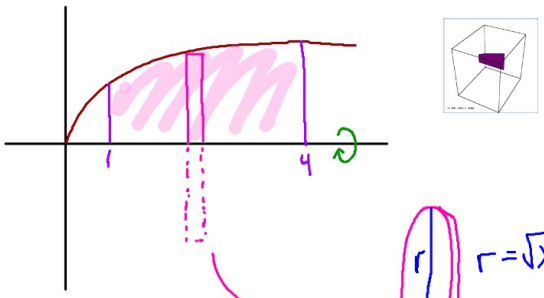
When finding the volume, what cross-section shapes will cause the answer to be a multiple of π? Why are those the only shapes?

Circles, (parts of circles)
ovals

October 30

Students will verbally explain how to find the volume using the disk method
(using the words:
axis of rotation, outside radius, inside radius...)

Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$, $x = 1$ and $x = 4$ about the x-axis



$r = \sqrt{x}$

$A = \pi r^2$

$A = \pi(\sqrt{x})^2 = \pi x$

$V_{slice} = A \cdot dx = \pi x \, dx$

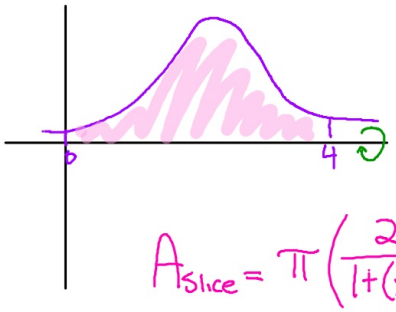
$$V_{total} = \int_1^4 \pi x \, dx = \pi \int_1^4 x \, dx$$

$$= \pi\left(\frac{x^2}{2}\Big|_1^4\right) = \pi\left(\frac{16}{2} - \frac{1}{2}\right) = \frac{15\pi}{2}$$

Find the volume of the solid generated by revolving the region bounded by

$$y = \frac{2}{1+(x-2)^2}$$

$x = 0$ and $x = 4$ about the x-axis



$$A_{slice} = \pi\left(\frac{2}{1+(x-2)^2}\right)^2$$

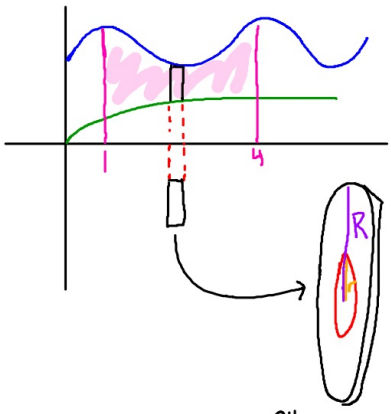
$$V_{slice} = \pi\left(\frac{2}{1+(x-2)^2}\right)^2 dx$$

$$V_{total} = \int_0^4 \pi\left(\frac{2}{1+(x-2)^2}\right)^2 dx$$

$$= \pi\int_0^4 \left(\frac{2}{1+(x-2)^2}\right)^2 dx$$

$$= 6.029\pi = 18.939$$

Find the volume of the solid generated by revolving the region bounded by
$y = \sin(x) + 4$
$y = \sqrt{x}$
$x = 1$ and $x = 4$
about the *x*-axis



$R = \sin x + 4$

$r = \sqrt{x}$

$$V_{outside} = \int_1^4 \pi (\sin x + 4)^2 \, dx = 59.0315\pi$$

$$V_{inside} = \int_1^4 \pi (\sqrt{x})^2 \, dx = \frac{15\pi}{2}$$

$$V_{total} = 59.0315\pi - \frac{15\pi}{2} = 161.891$$

Disk Method (Volumes of Revolution)

Slices are perpendicular to (perpendiskular) the axis of rotation

$$V = \int_a^b \pi r^2 \, dx = \pi \int_a^b r^2 \, dx$$

r = function

for 2 functions:

$$V_O - V_I = \int_a^b \pi R^2 \, dx - \int_a^b \pi r^2 \, dx$$

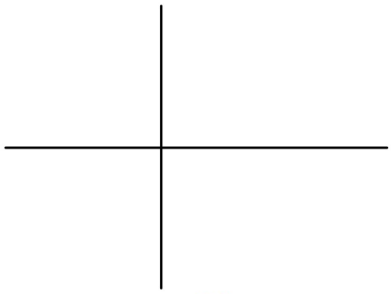
$$= \int_a^b \pi R^2 - \pi r^2 \, dx = \pi \int_a^b R^2 - r^2 \, dx$$

$$V = \pi \int_a^b \left(\text{outside radius}\right)^2 - \left(\text{inside radius}\right)^2 dx$$

outside radius (OR) - from the axis of rotation to the function that is farther away

inside radius (IR) - from the axis of rotation to the closer function

Find the volume
of the solid generated
by revolving the
region bounded by
$y = 2 + x(\cos(x))$
$y = \ln(x + 3)$
$x = -2$ and $x = 1$
about the *x*-axis



$$V = \pi \int_{-2}^{1} (2 + x\cos x)^2 - (\ln(x+3))^2 \, dx$$

$$= 30.916$$