October 9

How are anti-derivatives different from derivatives?

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Students will verbally explain how to evaluate definite integrals with u-substitution

(using the words: antiderivative, constant, substitution…)

$\int_2^4 (7x-3)^3\,dx$

Area between the curve $y=(7x-3)^3$, the x-axis, $x=2$ and $x=4$

$$\int_2^4 (7x-3)^3\,dx$$

$u=7x-3$

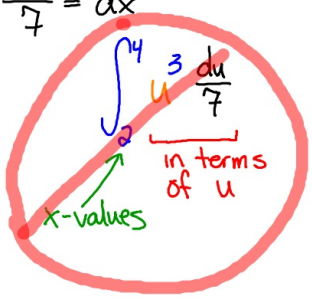
$du=7\,dx$

$\frac{du}{7}=dx$

Change bounds

if $x=2$, $u=7(2)-3=11$

if $x=4$, $u=7(4)-3=25$



$$\int_{11}^{25} u^3 \frac{du}{7} = \frac{1}{7}\int_{11}^{25} u^3\,du = \frac{1}{7}\left(\frac{u^4}{4}\right)\Big|_{11}^{25}$$

all in terms of u

$$=\frac{1}{7}\left(\frac{25^4}{4}-\frac{11^4}{4}\right)$$

$\int_0^4 2x\sqrt{16-x^2}\,dx$

$u=16-x^2$

$du=-2x\,dx$

$\frac{du}{-2x}=dx$

Change bounds

$x=4 \rightarrow u=16-4^2=0$

$x=0 \rightarrow u=16-0^2=16$

$$\int_{16}^{0} 2x\sqrt{u}\,\frac{du}{-2x}$$

$$\int_{16}^{0} -u^{1/2}\,du$$

flip bounds & multiply by a negative

$$=\int_0^{16} u^{1/2}\,du = \frac{u^{3/2}}{\frac{3}{2}}\Big|_0^{16}$$

$$=\frac{2}{3}u^{3/2}\Big|_0^{16}$$

$$=\frac{2}{3}(16)^{3/2}-\frac{2}{3}(0)^{3/2}$$

$$=\frac{2}{3}\left(\sqrt{16}\right)^3=\frac{2}{3}(64)=\frac{128}{3}$$