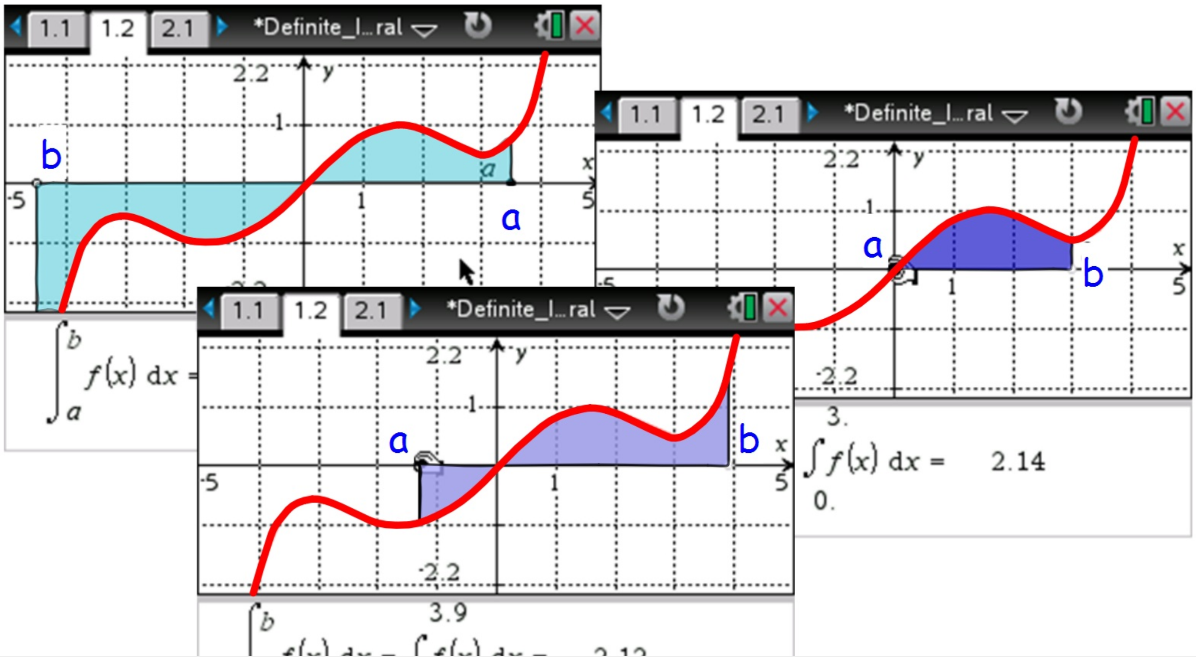
September 11

Explain why the definite integral of all three graphs below has the same value.



September 11

Students will verbally explain how to find the exact area under a curve using definite integrals

(using the words:
right, left, above, below, antiderivative...)

6. Based on your observations on pages 1.2 and 2.2, for any continuous function *f* on an interval [*c*, *d*] and for *a* and *b* in [*c*, *d*], when will the definite integral $\int_a^b f(x)\,dx$ be positive? Negative? Zero? Clearly explain your generalization.

Positive

- $b > a$ and $f(x) > 0$
- $b < a$ and $f(x) < 0$

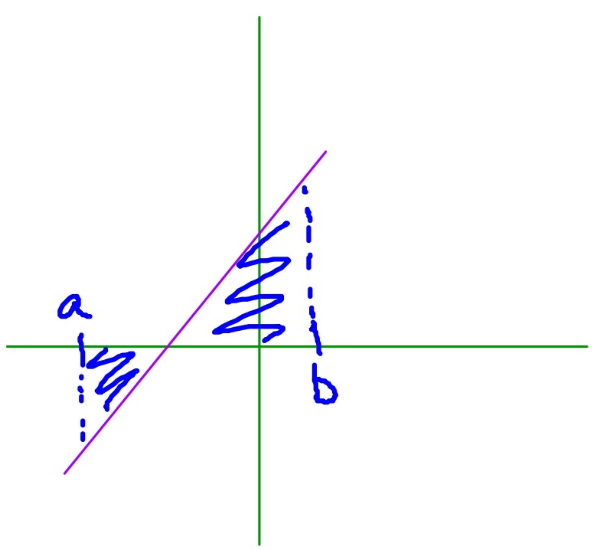
Negative

- $b < a$ and $f(x) > 0$
- $b > a$ and $f(x) < 0$

Zero

- if $a = b$
- if area above x-axis is equal to the area below the x-axis

7. The definite integral $\int_a^b f(x)\,dx$ is often described as "the area under the curve $y = f(x)$ between $x = a$ and $x = b$." What problems do you see with this definition?



Order of Integration	$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
Zero	$\int_a^a f(x)\,dx = 0$
Constant Multiple	$\int_a^b k \cdot f(x)\,dx = k\int_a^b f(x)\,dx$ For any number k
Additivity	$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$
Constant Multiple (special case)	$\int_a^b -f(x)\,dx = -\int_a^b f(x)\,dx \qquad k=-1$
Sum and Difference	$\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

Quotient Rule

Product Rule

Given

$\int_2^9 g(x)\,dx = -8$

$\int_2^4 f(x)\,dx = 7$

$\int_2^9 f(x)\,dx = -2$

$\int_4^9 f(x)\,dx = \int_2^4 f(x)dx + \int_4^9 f(x)dx = \int_2^9 f(x)\,dx$

$7 + \int_4^9 f(x)dx = -2$

$-7 \qquad\qquad -7$

$\int_4^9 f(x)dx = -9$

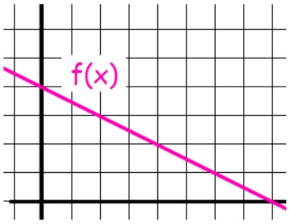
Find

$\int_2^9 4f(x)\,dx = 4\int_2^9 f(x)dx = 4(-2) = -8$

$\int_9^2 g(x)\,dx = -\int_2^9 g(x)dx = -(-8) = 8$

$\int_2^9 f(x) - g(x)\,dx = \int_2^9 f(x)\,dx - \int_2^9 g(x)dx = -2-(-8) = -2+8 = 6$

Is $\int_2^9 f(x)\cdot g(x)dx = 16$ Could be, but you don't know.



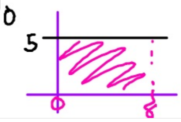
$$\int_0^8 f(x)\,dx = \frac{1}{2}(4)(8) = 16$$

$$\int_0^8 5f(x)\,dx = 5\int_0^8 f(x)\,dx = 5(16) = 80$$

$$\int_8^0 f(x)\,dx = -\int_0^8 f(x)\,dx = -16$$

$$\int_0^8 f(x)+5\,dx \neq 21 = 16+5$$

$$= \int_0^8 f(x)\,dx + \int_0^8 5\,dx = 16 + 8(5) = 56$$



$$\int_0^1 x^2\,dx = \int_0^1 x\,dx \cdot \int_0^1 x\,dx$$

$$\frac{1}{3} \neq \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$$