September 12

SWBAT:

Evaluate definite integrals using the Fundamental Theorem of Calculus

6. Drag point *a* on the top graph on page 1.4.

   a. What are you changing in the accumulation function when you change *a*? What are you changing in the graph of the accumulation function? Explain.

   b. Using what you know about the accumulation function, why do you think the bottom graph doesn't change when you change the value of *a*? Explain.

$$\int_{-3}^{x} f(t)\,dt \qquad \int_{-3}^{2} f(t)\,dt =$$

7. Suppose you are given that an accumulation function for a continuous function $f(x)$ can be expressed as $A(x) = x^2 + 3$. Explain how you can use this to find $\int_2^4 f(x)\,dx$.

$$\int_2^4 f(x)\,dx = A(4) - A(2) = 12$$

8. Based on your answers to questions 5 and 6, how do you think you would find a formula for an accumulation function of a continuous function without using the integral? Explain.

The anti-derivative

9. Using your response to question 8, describe how you would find the value of a definite integral for a continuous function *f*.

evaluate at the bounds + subtract (top-bottom)

10. Use your response to question 8 to find $\int_0^3 2x\,dx$. Explain your solution. How can you check your work?

$$\int_0^3 2x\,dx = x^2\Big|_0^3 = 3^2 - 0^2 = 9$$

$$\frac{d}{dx}(?) = 2x \qquad ? = x^2$$

Fundamental Theorem of Calculus

(part one)

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

where $F'(x) = f(x)$

$$\frac{d}{dx}\left(x^3\right) = 3x^2$$

$$\frac{d}{dx}\left(\sec(x)\right) = \sec(x)\tan(x)$$

$$\frac{d}{dx}\left(\ln(x)\right) = \frac{1}{x}$$

$$\frac{d}{dx}\left(\frac{x^6}{6}\right) = x^5$$

$$\frac{d}{dx}\left(2x^2 + 2x\right) = 4x + 2$$

$$\frac{d}{dx}\left(-e^{-x}\right) = e^{-x}$$

$$\frac{d}{dx}\left(6x\right) = 6$$

$$\frac{d}{dx}\left(-e^{-x}\right) = -e^{-x}(-1) = e^{-x}$$

$$\frac{d}{dx}\left(\frac{x^6}{6}\right) = 6\left(\frac{x^5}{6}\right) = x^5$$

$$\int_{-1}^{1} x^4\,dx$$

$$\frac{d}{dx}(?) = x^4$$

$$? = \frac{x^5}{5}$$

$$\int_{-1}^{1} x^4\,dx = \frac{x^5}{5}\Bigg|_{-1}^{1} = \frac{1^5}{5} - \left(\frac{(-1)^5}{5}\right) = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$$

$$\int_a^b x^n\,dx$$

$$\frac{d}{dx}(?) = x^n$$

$$? = \frac{x^{n+1}}{n+1}$$

$$\int_a^b x^n\,dx = \frac{x^{n+1}}{n+1}\Bigg|_a^b$$

$$= \frac{b^{n+1}}{n+1} - \frac{a^{n+1}}{n+1}$$

$$\int_3^{10} -\frac{2}{x}\,dx$$

$$= \int_3^{10} -2\left(\frac{1}{x}\right)dx = -2\int_3^{10} \frac{1}{x}\,dx$$

$$\frac{d}{dx}(?) = \frac{1}{x}$$

$$? = \ln(x)$$

$$-2\int_3^{10} \frac{1}{x}\,dx = -2\left(\ln x\Big|_3^{10}\right)$$

$$= -2\left(\ln(10) - \ln(3)\right)$$

$$= -2\left(\ln\left(\frac{10}{3}\right)\right)$$

$$\int_0^{1/4} \frac{1}{\sqrt{1-x^2}}\,dx$$

$$\frac{d}{dx}(?) = \frac{1}{\sqrt{1-x^2}}$$

$$? = \sin^{-1}(x)$$

$$\int_0^{1/4} \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x)\Big|_0^{1/4}$$

$$= \sin^{-1}\left(\frac{1}{4}\right) - \sin^{-1}(0)$$

$$= \sin^{-1}\left(\frac{1}{4}\right) - 0 = \sin^{-1}\left(\frac{1}{4}\right)$$

$$\int_4^9 \frac{1}{\sqrt{x}}\,dx$$

$$\int_4^9 x^{-1/2}\,dx$$

$$\frac{d}{dx}(?) = x^{-1/2}$$

$$? = \frac{x^{1/2}}{\frac{1}{2}} = 2x^{1/2}$$

$$\int_4^9 \frac{1}{\sqrt{x}}\,dx = 2x^{1/2}\Big|_4^9$$

$$= 2(9^{1/2}) - 2(4^{1/2})$$

$$= 2(3) - 2(2) = 2$$

$$2x^2 = 32$$

$$x = \pm 4$$