Tuesday, September 17

t	1	3	5	6	10	11
v(t)	8	-4	3	0	4	5

Given the table above, what information can you find (or approximate) about the acceleration? What information can you find (or approximate) about the distance traveled?

September 17

Students will verbally explain how to find the exact area under a curve using definite integrals

(using the words:
right, left, above, below, antiderivative...)

3. Without changing the value of a, use the accumulation function and your thinking from question 2 to find the following. For each, be sure to explain your thinking.

a. $\int_{1}^{4} f(t)dt =$ -6 $= \int_{-3}^{4} f(x)dx - \int_{-3}^{1} f(x)dx$

$A(4) - A(1)$

b. $\int_{-2}^{2} f(t)dt =$ ____________

c. $\int_{0}^{-1} f(t)dt =$ $.267$ $= -\left(\int_{-3} f(x)dx - \int_{-3} f(x)dx\right)$

$-(A(0) - A(-1))$

$\int_{-3}^{-1} f(x)\,dx - \int_{-3}^{0} f(x)dx$

$A(-1) - A(0)$

$$\int_{-3}^{x} f(x)\,dx = A(x)$$

5. The top graph on page 1.4 is the graph of the accumulation function, $y = A(x)$, for the function f from the previous pages, and the bottom graph shows the graph of its derivative, $y = A'(x)$.

a. Choose several values of x and find the corresponding values of $A'(x)$. For each of these, how do they compare to the value of $f(x)$ for that x? What do you observe? Does this make sense? Explain.

b. Given your response to a, complete the following:

$f(x)$ is derivative of $A(x)$.

$A(x)$ is antiderivative of $f(x)$.

7. Suppose you are given that an accumulation function for a continuous function $f(x)$ can be expressed as $A(x) = x^2 + 3$. Explain how you can use this to find $\int_2^4 f(x)\,dx$.

$$A(4) - A(2) = \int_2^4 f(x)\,dx$$

Fundamental Theorem of Calculus

(part one)

$$\boxed{\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)}$$

($f(x)$ is continuous)

where $F'(x) = f(x)$

evaluated from a to b

8. Based on your answers to questions 5 and 6, how do you think you would find a formula for an accumulation function of a continuous function without using the integral? Explain.

9. Using your response to question 8, describe how you would find the value of a definite integral for a continuous function *f*.

10. Use your response to question 8 to find $\int_0^3 2x\,dx$. Explain your solution. How can you check your work?

$$\int_0^3 2x\,dx = x^2\Big|_0^3 = 3^2 - 0^2 = 9$$

$$\frac{d}{dx}(?) = 2x$$

$$? = x^2$$

$$\frac{d}{dx}\left(x^3 \right) = 3x^2$$

$$\frac{d}{dx}\left(\sec(x) \right) = \sec(x)\tan(x)$$

$$\frac{d}{dx}\left(\ln(x) \right) = \frac{1}{x}$$

$$\frac{d}{dx}\left(\frac{x^6}{6} \right) = x^5$$

$$\frac{d}{dx}\left(2x^2 + 2x \right) = 4x + 2$$

e^{-x}

$$\frac{d}{dx}\left(-e^{-x} \right) = e^{-x}$$

$$\frac{d}{dx}\left(6x \right) = 6$$

$$\frac{d}{dx}\left(e^{-x}\right) = e^{-x}(-1)$$

$$\int_{-1}^{1} x^4\,dx$$

$$\frac{d}{dx}(?) = x^4$$

$$? = \frac{x^5}{5}$$

$$\int_{-1}^{1} x^4\,dx = \frac{x^5}{5}\Big|_{-1}^{1} = \frac{1^5}{5} - \frac{(-1)^5}{5} = \frac{1}{5} - \frac{-1}{5} = \frac{2}{5}$$

$$\int_a^b x^n\,dx$$

$$\frac{d}{dx}(?) = x^n$$

$$? = \frac{x^{n+1}}{n+1}$$

$$\int_a^b x^n\,dx = \frac{x^{n+1}}{n+1}\Big|_a^b$$

$$= \frac{b^{n+1}}{n+1} - \frac{a^{n+1}}{n+1}$$

$$\int_3^{10} \frac{-2}{x}\,dx$$

$$= \int_3^{10} -2\left(\frac{1}{x}\right)dx = -2\int_3^{10} \frac{1}{x}\,dx$$

$$\frac{d}{dx}(?) = \frac{1}{x}$$

$$? = \ln x$$

$$\int_3^{10} \frac{-2}{x}\,dx = -2\int_3^{10} \frac{1}{x}\,dx$$

$$= -2\left(\ln x\Big|_3^{10}\right)$$

$$= -2\left(\ln(10) - \ln(3)\right) = -2\ln\left(\frac{10}{3}\right)$$

$\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\,dx$

$\frac{d}{dx}(?) = \frac{1}{\sqrt{1-x^2}}$

$? = \sin^{-1}(x)$

$\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x)\Big|_0^{\frac{1}{2}}$

$= \sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(0)$

$= \frac{\pi}{6} - 0 = \frac{\pi}{6}$

Sets 5b

+ 6