September 25

If you know the rate something is changing, how does the integral help you find the total amount?

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Students will verbally explain how to solve problems using derivatives and integrals

(using the words:
rate, amount, integral…)

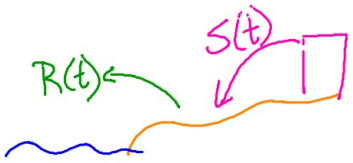
$$\int_a^b f'(x)\,dx = f(x)\Big|_a^b = f(b) - f(a)$$

derivative → Integral of the rate of change $\left(\int_a^b f'(x)\,dx\right)$ is equal to the net change $(f(b)-f(a))$ ← function

The tide removes sand from Sandy Point Beach at a rate modeled by the function R, given by
derivative → $R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right)$. removing sand

A pumping station adds sand to the beach at a rate modeled by the function S, given by
derivative → $S(t) = \frac{15t}{1+3t}$. adding sand

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.
rate = yd^3/hr t = hours



(0, 2500)
(hrs, yd^3)
(time, Amount of Sand)

(a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.
Amount $t = 0 \to 6$

$$\int_0^6 R(t)\,dt = \int_0^6 2 + 5\sin\left(\frac{4\pi t}{25}\right)dt = 31.816 \text{ yd}^3$$

(b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t.
Amount bound = t

$$Y(t) = \int_0^t S(x) - R(x)\,dx + 2500$$

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$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function S, given by

$$S(t) = \frac{15t}{1+3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

(c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

rate, at 4

$S(4) - R(4) = -1.909$ yd^3/hr

$S(t) - R(t)\Big|_{t=4}$

(d) For $0 \le t \le 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

Domain

$t = ?$ derivative of Amount $= 0$

$$\frac{d}{dx}\left(\int_0^x S(t) - R(t)\,dt + 2500\right)$$

$$= [S(x) - R(x)](1) - [S(0) - R(0)](0) + 0$$

$$= S(x) - R(x)$$

$$0 = S(x) - R(x) \rightarrow S(x) = R(x)$$

$x = 5.118$