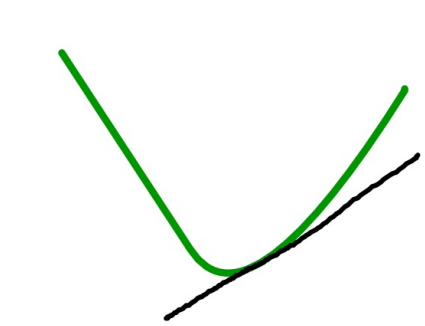
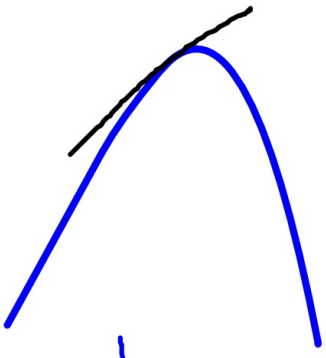
f''(x) > 0 is your tangent line an over or under estimate?





Underestimate because f'' > 0 so the function is concave up.

The tide removes sand from Sandy Point Beach at a rate modeled by the function R, given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function S, given by

$$S(t) = \frac{15t}{1+3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

(a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

(a) $\int_0^6 R(t)\,dt = 31.815$ or 31.816 yd^3

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer with units} \end{cases}$

(b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t.

(b) $Y(t) = 2500 + \int_0^t (S(x) - R(x))\,dx$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

$$\int_0^x S(t)\,dt + 2500 - \int_0^x R(t)\,dt$$

$$\int_0^x S(t) - R(t)\,dt + 2500$$

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(c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

(c) $Y'(t) = S(t) - R(t)$

$Y'(4) = S(4) - R(4) = -1.908$ or -1.909 yd^3/hr

1 : answer

$$\frac{d}{dx}\int_0^x S(t) - R(t)\,dt + 2500 = 1(S(x) - R(x))$$

(d) For $0 \le t \le 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

(d) $Y'(t) = 0$ when $S(t) - R(t) = 0$.
The only value in $[0, 6]$ to satisfy $S(t) = R(t)$ is $a = 5.117865$.

t	$Y(t)$
0	2500
a	2492.3694
6	2493.2766

The amount of sand is a minimum when $t = 5.117$ or 5.118 hours. The minimum value is 2492.369 cubic yards.

$3 : \begin{cases} 1 : \text{sets } Y'(t) = 0 \\ 1 : \text{critical } t\text{-value} \\ 1 : \text{answer with justification} \end{cases}$

$Y_1 = S(t) - R(t)$

$Y_2 = 0$

$$\int_0^{5.117865} S(t) - R(t)\,dt + 2500 = 2492.369$$

approximate area using LRAM, RRAM, MRAM

evaluate definite integrals using basic shapes to find the area

evaluate definite integrals using the rules of definite integrals

calculate the definite integral using the FTC

calculate the derivative of an integral using the FTC

Apply Definite Integrals