Tuesday, September 3

What types of functions have derivatives that will eventually become constant? What types of functions have derivatives that will eventually return to the same function (or the same function multiplied by a constant)?
Explain your reasoning.

September 3

Students will verbally explain how to compare the growth rates of different functions
(using the words:
limit, infinity, L'Hopital's Rule...)

$\lim_{x\to\infty}\frac{f(x)}{g(x)}=0$	$\lim_{x\to\infty}\frac{f(x)}{g(x)}=\infty$	$\lim_{x\to\infty}\frac{f(x)}{g(x)}=C$ (C is a constant other than 0)
$\lim_{x\to\infty}\frac{\sqrt{x}}{e^x}$	$\lim_{x\to\infty}\frac{4x^3+1}{x^2}$	$\lim_{x\to\infty}\frac{16x^5-2x^2}{4x^5+3x}$
$\lim_{x\to\infty}\frac{\sqrt{x}}{x^2}$	$\lim_{x\to\infty}\frac{e^x}{x^2}$	$\lim_{x\to\infty}\frac{4x^4+12}{15x-3x^4}$
$\lim_{x\to\infty}\frac{\ln(x)}{e^x}$	$\lim_{x\to\infty}\frac{3x^4-2}{\sqrt[3]{x}}$	$\lim_{x\to\infty}\frac{e^x-x}{5e^x}$
$\lim_{x\to\infty}\frac{5\ln(x)}{6+3x^3}$	$\lim_{x\to\infty}\frac{e^{3x}}{5x^3-9x}$	$\lim_{x\to\infty}\frac{8+\ln(x)}{\ln(x^3)}$
$\lim_{x\to\infty}\frac{7\ln(x)}{8x}$	$\lim_{x\to\infty}\frac{e^{x^2}}{\ln(3x)}$	

Relative Growth Rates of functions

fastest ⟶ slowest

$e^x > x^n > \ln(x)$

factorials $n!$ $(5!=5\cdot4\cdot3\cdot2\cdot1)$ > exponential functions > power functions > log functions

$\lim_{x\to\infty}\frac{f(x)}{g(x)}$

- if $\lim_{x\to\infty}\frac{f(x)}{g(x)}=0$ then $g(x)$ grows faster
- if $\lim_{x\to\infty}\frac{f(x)}{g(x)}=\infty$ then $f(x)$ grows faster
- if $\lim_{x\to\infty}\frac{f(x)}{g(x)}=C$ the $f(x)$ + $g(x)$ grow at the same rate

Which function grows faster?

$500x^{600}$ or e^x

e^x grows faster

$$\lim_{x\to\infty} \frac{e^x}{500x^{600}} = \infty$$

5^x or 2^x

5^x grows faster

$$\lim_{x\to\infty} \frac{5^x}{2^x} = \lim_{x\to\infty} \left(\frac{5}{2}\right)^x = \infty$$

$\ln x$ or e^{-x}

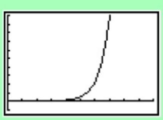
$\ln x$ grows faster

e^{-x} is decreasing

$$\lim_{x\to\infty} \frac{f(x)}{g(x)}$$

f(x) grows faster

X	Y
1	2
10	1024
100	1.27E+30
1000	1.1E+301
10000	error



$$\lim_{x\to\infty} \frac{x^5}{x^3+5000} \qquad \lim_{x\to\infty} \frac{4^x}{x^4}$$

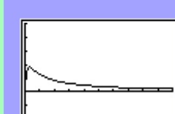
$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \infty$$

g(x) grows faster

$$\lim_{x\to\infty} \frac{\ln(4x)}{x^4}$$

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$$

X	Y
1	0.25
10	9.54E-07
100	6.22E-61
1000	error
10000	error



$$\lim_{x\to\infty} \frac{4^{2x}}{4^{5x}}$$

f(x) and g(x) grow at the same rate

X	Y
1	8
10	5.3
100	5.03
1000	5.003
10000	5.0003

$$\lim_{x\to\infty} \frac{100x^2+7x+1000}{5x^2}$$

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = 5$$

$$\frac{d}{dx}\left(\log_b x\right) = \frac{1}{x \cdot \ln b}$$