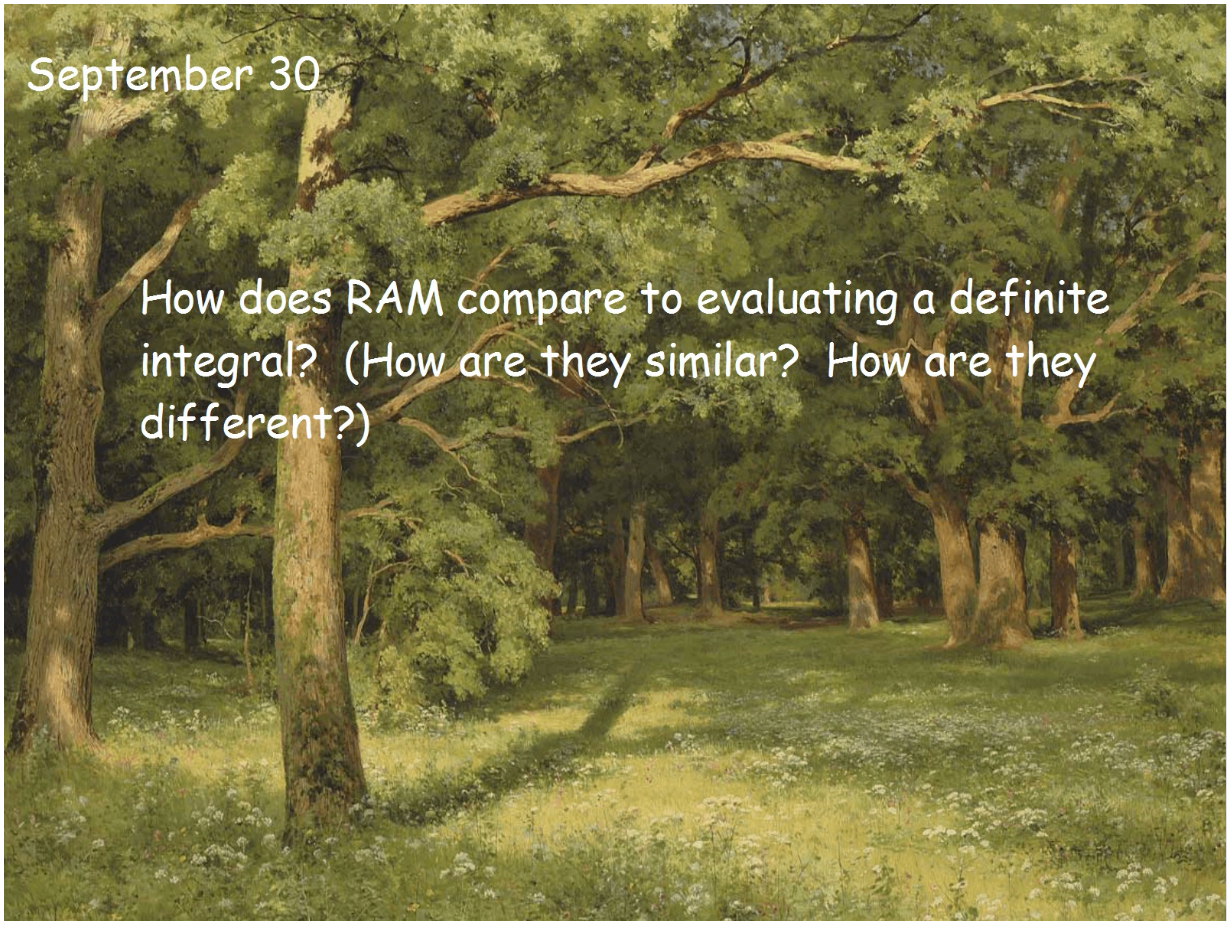


September 30

Students will verbally explain how to approximate the area under a curve

(using the words:
base, height, y-value...)

- ☺ RAM - including TAM
- ☺ Properties of Definite Integrals
- ☺ Evaluating Definite Integrals using areas of basic shapes
- ☺ Evaluating Definite Integrals using the FTC
- ☺ Taking the derivative of an integral - FTC 2
- ☺ Integral as an Accumulator $\int \text{rate} = \text{Amount}$
  (like Sandy Beach and Amusement Park Problem)

Sets #3 - 9 (corrections 5-9)

2. The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{\left(t^2 - 24t + 160\right)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{\left(t^2 - 38t + 370\right)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round your answer to the nearest whole number.

(b) The price of admission to the park is $15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is $11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

Copyright © 2002 by College Entrance Examination Board. All rights reserved.
Advanced Placement Program and AP are registered trademarks of the College Entrance Examination Board.

AP® CALCULUS AB 2002 SCORING GUIDELINES

Question 2

The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{(t^2 - 24t + 160)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{(t^2 - 38t + 370)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round answer to the nearest whole number.

(b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

(c) Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the park.

(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

(a) $\boxed{\int_9^{17} E(t)\,dt} = 6004.270$

$\boxed{6004}$ people entered the park by 5 pm.

3 { 1 : limits; 1 : integrand; 1 : answer

(b) $15\int_9^{17} E(t)\,dt + 11\int_{17}^{23} E(t)\,dt = 104048.165$

The amount collected was \$104,048.

or

$\int_{17}^{23} E(t)\,dt = 1271.283$

1271 people entered the park between 5 pm and 11 pm, so the amount collected was

$\$15 \cdot (6004) + \$11 \cdot (1271) = \$104,041.$

1 : setup

(c) $H'(17) = E(17) - L(17) = \boxed{-380.281}$

There were 3725 people in the park at $t = 17$.

The number of people in the park was decreasing at the rate of approximately 380 people/hr at time $t = 17$.

3 { 1 : value of $H'(17)$; 2 : meanings — 1 : meaning of $H(17)$; 1 : meaning of $H'(17)$; < −1 > if no reference to $t = 17$

(d) $H'(t) = \boxed{E(t) - L(t) = 0}$ $H'(t) = 0$

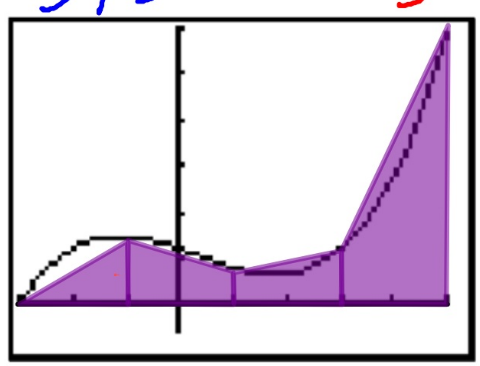
$t = 15.794$ or 15.795

2 { 1 : $E(t) - L(t) = 0$; 1 : answer

Copyright © 2002 by College Entrance Examination Board. All rights reserved.
Advanced Placement Program and AP are registered trademarks of the College Entrance Examination Board.

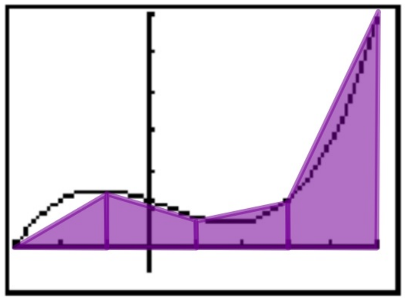
LRAM $34\frac{2}{3}$

RRAM $93\frac{1}{3}$



Average of LRAM + RRAM = 64

$$f(x) = \frac{x^3}{3} - \frac{x^2}{3} - 2x + 6$$



Int	Y_1-value	Y_2-value	width	Area $= \frac{1}{2}(w)(Y_1$
$-3 \to -1$	0	7.333	2	7.333
$-1 \to 1$	7.333	4	2	11.333
$1 \to 3$	4	6	2	10
$3 \to 5$	6	29.333	2	35.333
			Total Area	64

Trapezoid Rule

When you have even intervals...

$$A_T = \frac{LRAM + RRAM}{2}$$

$$A_T = \frac{1}{2}(h)(y_1 + 2y_2 + 2y_3 + \cdots + 2y_{n-1} + y_n)$$

width of intervals

Over estimate when the function is concave up ++

under estimate when the function is concave down --

Use 4 trapezoids of equal width to approximate $\int_1^3 x^2+6\,dx$

Int	Y_1	Y_2	width	Area
$1\to\frac{3}{2}$	7	8.25	$\frac{1}{2}$	3.813
$\frac{3}{2}\to 2$	8.25	10	$\frac{1}{2}$	4.563
$2\to\frac{5}{2}$	10	12.25	$\frac{1}{2}$	5.563
$\frac{5}{2}\to 3$	12.25	15	$\frac{1}{2}$	6.813
			Total Area	20.752

$$\text{width}=\frac{3-1}{4}=\frac{1}{2}$$

$$\frac{1}{2}\left(\frac{1}{2}\right)\left(7+2(8.25)+2(10)+2(12.25)+15\right)$$

$$=20.75$$

x	f(x)
-3	7
0	29
1	6
4	2
9	8

			Total Area	

Use 4 trapezoids to approximate $\int_{-3}^{9} f(x)\,dx$