Monday, September 9

LRAM, RRAM, and MRAM give estimates of the area under a curve. Which one gives the best estimate? Why? When do you have an over-estimate? An under-estimate?

September 9

Students will verbally explain how to estimate and find the exact area under a curve

(using the words:
right, left, midpoint, area, bounds, exact, approximation...)

$f(x) = x^2 + 2$

a. Consider the function $F(x) = \frac{x^3}{3} + 2x$. Compute $F(6) - F(1)$. How does this value compare to the exact answer in questions 1, 2, and 3? What is the relationship between F and f?

$f(x) = F'(x)$

$F(6) = 84$

$F(1) = 2\frac{1}{3}$

$F(6) - F(1) = 81\frac{2}{3}$

same as the exact area

b. Consider the function $f1(x) = -x^2 + 36$ on the interval [1, 6]. Compute *LRAM*, *RRAM*, and *MRAM* for various values of n. Explain what happens to these values as n increases. Why is *LRAM* always an overestimate and why is *RRAM* always an underestimate?

c. Consider the function $f1(x) = 3x^2 - 18x + 32$ on the interval [1, 6]. Compute *LRAM*, *RRAM*, and *MRAM* for various values of n. Explain what happens to these values as n increases. Is it possible to predict whether *LRAM* or *RRAM* will always be an underestimate or overestimate? Why or why not?

Riemann Sum

approximate area between the curve and the x-axis

$$\sum_{k=1}^{n} f(x_k)\,\Delta x_k$$

Exact Area

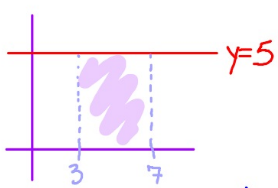
$$\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\,\Delta x_k$$

$$= \int_a^b f(x)\,dx$$

= exact area between f(x) and the x-axis from x=a to x=b

"Integral of f(x) from a to b"

$$\int_3^7 5\,dx$$



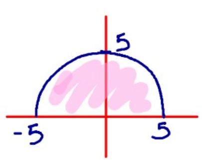
Rectangle

Area = b·h

A = 4·5 = 20

$$\int_3^7 5\,dx = 20$$

$$\int_{-5}^{5} \sqrt{25-x^2}\,dx$$



Semicircle

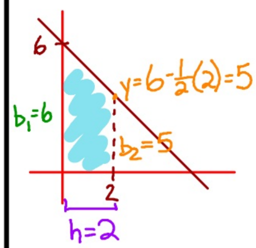
Area = $\frac{1}{2}\pi r^2$

$A = \frac{1}{2}\pi(5)^2$

$A = \frac{25}{2}\pi$

$$\int_{-5}^{5} \sqrt{25-x^2}\,dx = \frac{25\pi}{2}$$

$$\int_0^2 6 - \frac{1}{2}x \, dx$$



$$\int_0^2 6 - \frac{1}{2}x \, dx = 11$$

Trapezoid

$$\text{Area} = \frac{b_1+b_2}{2}\cdot h = \frac{h(b_1+b_2)}{2}$$

$$A = \frac{6+5}{2}\cdot 2 = 11$$