April 21

How is the area under a curve related to the slope of the curve?

$$\text{Area} = \int f(x)\,dx = F(x)$$

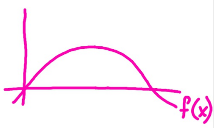
$$\text{Slope} = \frac{d}{dx}(f(x)) = F''(x)$$

April 21

Students will verbally explain how to solve problems using calculus

(using the words: integral, derivative, slope, etc...)

$g(x) = \int_0^x f(t)\,dt$



Given a graph of f(x):

How do you find the value of g(a)?

find the area between the curve + the x-axis from 0 to a

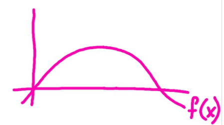
How do you find the value of g'(a)?

the y-value on the graph of f(x) at $x=a$

How do you find the value of g"(a)?

slope of f(x) at $x=a$

$g(x) = \int_0^x f(t)\,dt$



Given a graph of f(x):

How do you find where a maximum of g(x) occurs?

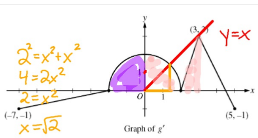
where $f(x)=0$ (or is undefined) and changes from positive to negative

How do you find where a minimum of g(x) occurs?

where $f(x)=0$ (or is undefined) and changes from negative to positive

How do you find where an inflection point occurs?

max or min of f(x)



$\int_0^3 g'(x)\,dx = g(3)-g(0)$

$\frac{1}{4}\pi(2)^2+\frac{3}{2} = g(3)-5$

$\pi+\frac{3}{2}+5=g(3)=\pi+6.5$

$\int_{-2}^0 g'(x)\,dx = g(0)-g(-2)$

$\pi = 5-g(-2)$

$g(-2)=5-\pi$

5. The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find $g(3)$ and $g(-2)$.

(b) Find the x-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.

$x=0,2,3$ b/c the slope of $g'(x)$ changes sign

b/c $g'(x)$ has a min or max

(c) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of each critical point of h, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

$h'(x)=g'(x)-x$

$0=g'(x)-x$

$x=g'(x)$

$x=3, \sqrt{2}$

CP		$\sqrt{2}$	3
sign h'	+	−	−
behav h	inc	dec	dec

max at $x=\sqrt{2}$ because $h'(x)$ changes from pos to neg

neither max or min at $x=3$ because $h'(x)$ does not change signs