April 22

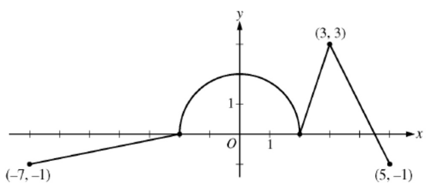
For time $t \geq 0$, the position of a particle traveling along a line is given by a differentiable function s. If s is increasing for $0 \leq t < 2$ and s is decreasing for $t > 2$, what is the total distance the particle travels for $0 \leq t \leq 5$?

$$\int_0^2 s'(x)\,dx - \int_2^5 s'(x)\,dx$$

$$\int_0^5 |s'(t)|\,dt$$

April 22

Students will verbally explain how to solve problems using calculus (using the words: integral, derivative, slope, etc...)



Graph of g'

5. The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure above.

   (a) Find $g(3)$ and $g(-2)$.

   (b) Find the x-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.

   (c) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of each critical point of h, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

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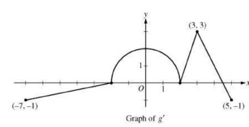
Question 5



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(a) Find $g(3)$ and $g(-2)$.

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(c) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of each critical point of h, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

(a) $g(3) = 5 + \int_0^3 g'(x)\,dx = 5 + \frac{\pi \cdot 2^2}{4} + \frac{3}{2} = \frac{13}{2} + \pi$

$g(-2) = 5 + \int_0^{-2} g'(x)\,dx = 5 - \pi$

$3 : \begin{cases} 1 : \text{uses } g(0) = 5 \\ 1 : g(3) \\ 1 : g(-2) \end{cases}$

(b) The graph of $y = g(x)$ has points of inflection at $x = 0$, $x = 2$, and $x = 3$ because g' changes from increasing to decreasing at $x = 0$ and $x = 3$, and g' changes from decreasing to increasing at $x = 2$.

$2 : \begin{cases} 1 : \text{identifies } x = 0, 2, 3 \\ 1 : \text{explanation} \end{cases}$

(c) $h'(x) = g'(x) - x = 0 \Rightarrow g'(x) = x$

On the interval $-2 \leq x \leq 2$, $g'(x) = \sqrt{4 - x^2}$.

On this interval, $g'(x) = x$ when $x = \sqrt{2}$.

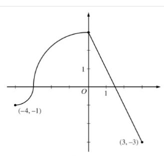
The only other solution to $g'(x) = x$ is $x = 3$.

$h'(x) = g'(x) - x > 0$ for $0 \leq x < \sqrt{2}$

$h'(x) = g'(x) - x \leq 0$ for $\sqrt{2} < x \leq 5$

Therefore h has a relative maximum at $x = \sqrt{2}$, and h has neither a minimum nor a maximum at $x = 3$.

$4 : \begin{cases} 1 : h'(x) \\ 1 : \text{identifies } x = \sqrt{2}, 3 \\ 1 : \text{answer for } \sqrt{2} \text{ with analysis} \\ 1 : \text{answer for 3 with analysis} \end{cases}$



Graph of f

4. The continuous function f is defined on the interval $-4 \leq x \leq 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above. Let $g(x) = 2x + \int_0^x f(t)\,dt$.

   (a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

   (b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \leq x \leq 3$. Justify your answer.

   (c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

   (d) Find the average rate of change of f on the interval $-4 \leq x \leq 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

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Question 4

The continuous function f is defined on the interval $-4 \leq x \leq 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above.

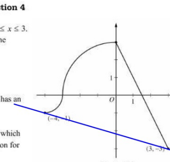
Let $g(x) = 2x + \int_0^x f(t)\,dt$.

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

(b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \leq x \leq 3$. Justify your answer.

(c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

(d) Find the average rate of change of f on the interval $-4 \leq x \leq 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.



Graph of f

(a) $g(-3) = 2(-3) + \int_0^{-3} f(t)\,dt = -6 - \frac{9\pi}{4}$

$g'(x) = 2 + f(x)$

$g'(-3) = 2 + f(-3) = 2$

$3 : \begin{cases} 1 : g(-3) \\ 1 : g'(x) \\ 1 : g'(-3) \end{cases}$

(b) $g'(x) = 0$ when $f(x) = -2$. This occurs at $x = \frac{5}{2}$.

$g'(x) > 0$ for $-4 < x < \frac{5}{2}$ and $g'(x) < 0$ for $\frac{5}{2} < x < 3$.

Therefore g has an absolute maximum at $x = \frac{5}{2}$.

$3 : \begin{cases} 1 : \text{considers } g'(x) = 0 \\ 1 : \text{identifies interior candidate} \\ 1 : \text{answer with justification} \end{cases}$

(c) $g''(x) = f'(x)$ changes sign only at $x = 0$. Thus the graph of g has a point of inflection at $x = 0$.

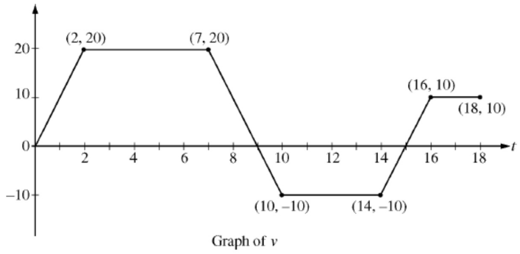
1 : answer with reason

(d) The average rate of change of f on the interval $-4 \leq x \leq 3$ is $\frac{f(3) - f(-4)}{3 - (-4)} = -\frac{2}{7}$.

To apply the Mean Value Theorem, f must be differentiable at each point in the interval $-4 < x < 3$. However, f is not differentiable at $x = -3$ and $x = 0$.

$2 : \begin{cases} 1 : \text{average rate of change} \\ 1 : \text{explanation} \end{cases}$

$$\lim_{h \to 0^+} \frac{f(x+h) - f(x)}{h}$$



Graph of v

4. A squirrel starts at building A at time $t = 0$ and travels along a straight, horizontal wire connected to building B. For $0 \le t \le 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.

   (a) At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.

   (b) At what time in the interval $0 \le t \le 18$ is the squirrel farthest from building A? How far from building A is the squirrel at that time?

   (c) Find the total distance the squirrel travels during the time interval $0 \le t \le 18$.

   (d) Write expressions for the squirrel's acceleration $a(t)$, velocity $v(t)$, and distance $x(t)$ from building A that are valid for the time interval $7 < t < 10$.

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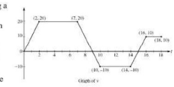
Question 4

A squirrel starts at building A at time $t = 0$ and travels along a straight wire connected to building B. For $0 \le t \le 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.



(a) At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.

(b) At what time in the interval $0 \le t \le 18$ is the squirrel farthest from building A? How far from building A is the squirrel at this time?

(c) Find the total distance the squirrel travels during the time interval $0 \le t \le 18$.

(d) Write expressions for the squirrel's acceleration $a(t)$, velocity $v(t)$, and distance $x(t)$ from building A that are valid for the time interval $7 < t < 10$.

(a) The squirrel changes direction whenever its velocity changes sign. This occurs at $t = 9$ and $t = 15$.

$2 : \begin{cases} 1 : t\text{-values} \\ 1 : \text{explanation} \end{cases}$

(b) Velocity is 0 at $t = 0$, $t = 9$, and $t = 15$.

t	position at time t
0	0
9	$\frac{9+5}{2} \cdot 20 = 140$
15	$140 - \frac{6+4}{2} \cdot 10 = 90$
18	$90 + \frac{3+2}{2} \cdot 10 = 115$

The squirrel is farthest from building A at time $t = 9$; its greatest distance from the building is 140.

$2 : \begin{cases} 1 : \text{identifies candidates} \\ 1 : \text{answers} \end{cases}$

(c) The total distance traveled is $\int_0^{18} |v(t)|\,dt = 140 + 50 + 25 = 215$.

1 : answer

(d) For $7 < t < 10$, $a(t) = \frac{20 - (-10)}{7 - 10} = -10$

$v(t) = 20 - 10(t - 7) = -10t + 90$

$x(7) = \frac{7+5}{2} \cdot 20 = 120$

$$x(t) = x(7) + \int_7^t (-10u + 90)\,du = 120 + \left(-5u^2 + 90u\right)\Big|_{u=7}^{u=t} = -5t^2 + 90t - 265$$

$4 : \begin{cases} 1 : a(t) \\ 1 : v(t) \\ 2 : x(t) \end{cases}$