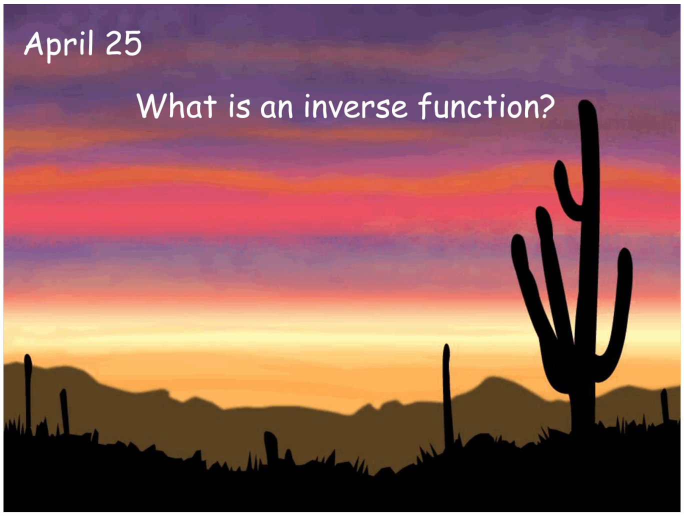

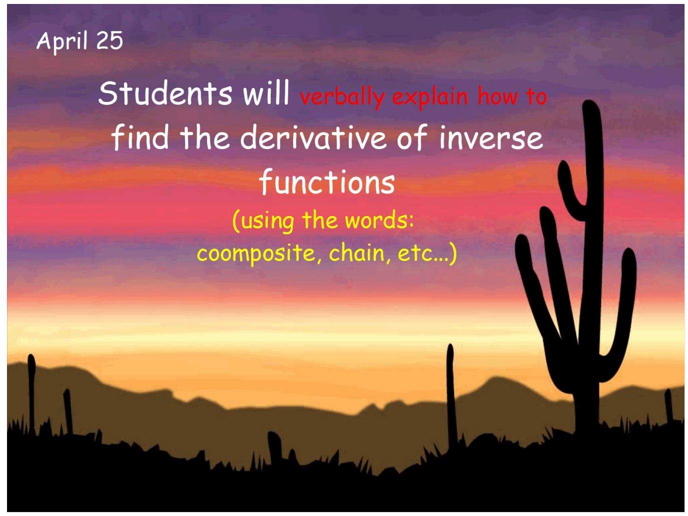

Inverse functions

If f(x) and g(x) are inverse functions:

$f(g(x)) = x$ range for $f(x)$ = domain for $g(x)$

Find g'(x) - the derivative of $f^{-1}(x)$

$f'(g(x)) \cdot g'(x) = 1$

$(f^{-1})'(x) = g'(x) = \dfrac{1}{f'(g(x))}$

U. Derivative of an Inverse of a Function

What you are finding: There is probably no topic that confuses students (and teachers) more than inverses. The inverse of a function *f* is another function f^{-1} that "undoes" what *f* does. So $f^{-1}(f(x)) = x$. For instance, the inverse of adding 5 is subtracting 5. Start with any number *x*, add 5, then subtract 5, and you are back to *x*. Do not confuse the inverse f^{-1} with the reciprocal. $x^{-1} = \frac{1}{x}$ but $(f(x))^{-1} \neq \frac{1}{f(x)}$.

To find the inverse of a function, you replace *x* with *y* and *y* with *x*. The inverse to the function $y = 4x - 1$ is $x = 4y - 1$ or $y = \frac{x+1}{4}$.

In this section, you are concerned with finding the derivative of the inverse to a function: $\frac{d}{dx}[f(x)]^{-1}$

How to find it: The formula used is: $\frac{dy}{dx} = \frac{1}{f'(y)}$. But what I suggest, rather than memorizing this formula, is to switch *x* and *y* to find the inverse, and then take the derivative, using implicit differentiation: $x = f(y) \Rightarrow 1 = f'(y)\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{f'(y)}$.

Example 72: Find the derivative of the inverse to $y = x^3$ at $x = 8$

$g(x) = f^{-1}(x)$

$\boxed{g'(x) = \dfrac{1}{f'(g(x))}}$

$g'(8) = \dfrac{1}{f'(g(8))}$

$g'(8) = \dfrac{1}{f'(2)}$

$= \dfrac{1}{3(2)^2} = \dfrac{1}{12}$

$f'(x) = 3x^2$

$g(8) \rightarrow 8 = f(x)$

$8 = x^3$

$x = 2$

$g(8) = 2$

Example 74: (Calc) Find the derivative of the inverse to $f(x) = x + 3\sin x$ at $x = 6$.

Example 75: If $f(x) = x^2$ (1st quadrant), write the equation of the tangent line to $f^{-1}(x)$ at $x = 16$.

Example 76: If $f(x) = x^3 + x^2 + x + 1$, write the equation of the tangent line to $f^{-1}(x)$ at $x = 4$.

Example 77: In the chart to the right, selected values of *x* are given along with values of $f(x)$ and $f'(x)$.

x	$f(x)$	$f'(x)$
-2	3	-1
-1	2	-2
0	-1	4
1	1	-3

a) If f^{-1} is the inverse function of *f*, find the derivative of f^{-1} at $x = -1$: $(f^{-1})'(-1)$.

$g(x) = f^{-1}(x)$ $g'(x) = \dfrac{1}{f'(g(x))}$

$g(-1) = 0$

$-1 = f(x)$

$g'(-1) = \dfrac{1}{f'(g(-1))} = \dfrac{1}{f'(0)} = \dfrac{1}{4}$

b) Write an equation for the line tangent to the graph of $y = f^{-1}(x)$ at $x = 2$.

$x = 2$

$y = g(2) = f^{-1}(2) = -1$

$(2 = f(x))$

slope $= \dfrac{1}{f'(g(2))} = \dfrac{1}{f'(-1)} = \dfrac{1}{-2}$

$y + 1 = -\frac{1}{2}(x - 2)$