February 10

How do you know if a series is a geometric series, p-series, or neither?

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Students will verbally explain how to determine if a series converges using the ratio test

(using the words:
term, ratio, simplify...)

Simplify:

$$\frac{3(n+1)!\,x^4}{5(n)!\,x^5}$$

$$\frac{2^{n+2}\,(n)!}{2^n\,(n+2)!} = \frac{\cancel{2^n}\,2^2\,\cancel{(n)}\cancel{(n-1)}\cancel{(n-2)}\cdots\cancel{(2)}\cancel{(1)}}{\cancel{2^n}\,(n+2)(n+1)\cancel{(n)}\cancel{(n-1)}\cancel{(n-2)}\cdots\cancel{(2)}\cancel{(1)}}$$

$$\frac{(n+1)\,x^{n+1}\,7^n}{7^{n+1}(n)\,x^n}$$

$\frac{3(n+1)!\,x^4}{5(n)!\,x^5}$	$\frac{2^2}{(n+2)(n+1)}$
$\frac{2^{n+2}\,(n)!}{2^n\,(n+2)!}$	$\frac{(n+1)\,x}{7(n)}$
$\frac{(n+1)\,x^{n+1}\,7^n}{7^{n+1}(n)\,x^n}$	$\frac{3(n+1)}{5x}$

$$\sum_{n=1}^{\infty} a_n \text{ is a series}$$

$$\text{and } \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L \quad (\rho)$$

If $L > 1$, the series diverges

If $L < 1$, the series converges

If $L = 1$, the test fails

Ratio Test

Determine if $\sum_{n=1}^{\infty} \frac{4^n}{5^n+1}$ converges, using the ratio test

$$a_{n+1} = \frac{4^{n+1}}{5^{n+1}+1} \qquad a_n = \frac{4^n}{5^n+1}$$

$$\lim_{n\to\infty}\left|\frac{\frac{4^{n+1}}{5^{n+1}+1}}{\frac{4^n}{5^n+1}}\right| = \lim_{n\to\infty}\left|\frac{4^{n+1}}{5^{n+1}+1}\cdot\frac{5^n+1}{4^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{4^{n+1}}{4^n}\cdot\frac{5^n+1}{5^{n+1}+1}\right|$$

$$= \lim_{n\to\infty}\left|4\cdot\frac{5^n+\cancel{1}}{5^{n+1}+\cancel{1}}\right|$$

$$= \lim_{n\to\infty}\left|4\cdot\frac{5^n}{5^{n+1}}\right| = \lim_{n\to\infty}\left|4\cdot\frac{1}{5}\right| = \frac{4}{5}$$

$\sum_{n=1}^{\infty} \frac{4^n}{5^n+1}$ converges by the Ratio Test $\left(\frac{4}{5} < 1\right)$

Determine if $\sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!}$ converges

$$a_n = \frac{(n!)^3}{(3n)!} \qquad a_{n+1} = \frac{((n+1)!)^3}{(3(n+1))!}$$

$$\lim_{n\to\infty}\left|\frac{\frac{((n+1)!)^3}{(3(n+1))!}}{\frac{(n!)^3}{(3n)!}}\right| = \lim_{n\to\infty}\left|\frac{((n+1)!)^3}{(3(n+1))!}\cdot\frac{(3n)!}{(n!)^3}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(3n)!}{(3(n+1))!}\cdot\frac{((n+1)!)^3}{(n!)^3}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(3n)!}{(3n+3)!}\cdot\frac{(n+1)!}{(n!)}\cdot\frac{(n+1)!}{n!}\cdot\frac{(n+1)!}{n!}\right|$$

$$= \lim_{n\to\infty}\left|\frac{\cancel{(3n)}\cancel{(3n-1)}\cancel{(3n-2)}\cdots\cancel{(2)}\cancel{(1)}}{(3n+3)(3n+2)(3n+1)\cancel{(3n)}\cdots\cancel{(2)}\cancel{(1)}}\cdot\frac{(n+1)\cancel{(n)}\cancel{(n-1)}\cdots\cancel{(2)}\cancel{(1)}}{\cancel{n}\cancel{(n-1)}\cdots\cancel{(2)}\cancel{(1)}}\cdot\frac{(n+1)!}{\cancel{n!}}\cdot\frac{(n+1)!}{\cancel{n!}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{1}{(3n+3)(3n+2)(3n+1)}\cdot\frac{(n+1)(n+1)(n+1)}{1}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(n+\cancel{1})(n+\cancel{1})(n+\cancel{1})}{(3n+\cancel{3})(3n+\cancel{2})(3n+\cancel{1})}\right| = \lim_{n\to\infty}\left|\frac{n^3}{27n^3}\right| = \frac{1}{27}$$

$\sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!}$ converges by the ratio test ($\frac{1}{27}<1$)

Pg 578

#1-20 (skip multiples of 3)