February 13

How do you find the limit when x is approaching infinity?

$$\lim_{x\to\infty} \frac{4x^2-10x^7}{x^7+1000x} = \lim_{x\to\infty} \frac{-10x^7}{x^7} = -10$$

Direct Comparison Test

February 12

Students will verbally explain how to determine if a series converges using the direct comparison test

(using the words: simplify, compare...)

Direct Comparison Test

$\sum a_n$ is a series with no negative terms

$a_n \le c_n$ $\quad a_n \ge d_n$

If $\sum c_n$ converges, then $\sum a_n$ converges

If $\sum d_n$ diverges, then $\sum a_n$ diverges

Direct Comparison Test

Use the Direct Comparison Test to determine if $\sum_{n=0}^{\infty} 2^{-n!}$ converges

$a_n = 2^{-n!} = \frac{1}{2^{n!}}$

compare to $c_n = \frac{1}{2^n} = \left(\frac{1}{2}\right)^n$

$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$ ← geometric $r = \frac{1}{2} < 1$

$\therefore \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$ converges

$\frac{1}{2^{n!}} \le \frac{1}{2^n}$

$\therefore \sum_{n=0}^{\infty} 2^{-n!}$ converges by DCT

Determine if $\sum_{n=0}^{\infty} \frac{x^{2n}}{(n!)^2}$ converges

$a_n = \frac{x^{2n}}{(n!)^2}$

compare to $b_n = \frac{x^{2n}}{n!}$

$\frac{x^{2n}}{(n!)^2} \le \frac{x^{2n}}{n!}$

$$\sum_{n=0}^{\infty} \frac{x^{2n}}{n!} = 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} + \cdots + \frac{(x^2)^n}{n!} + \cdots = e^{x^2}$$

$\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$ converges (to e^{x^2})

$\therefore \sum_{n=0}^{\infty} \frac{x^{2n}}{(n!)^2}$ converges

Determine if $\sum_{n=3}^{\infty} \frac{\ln(n)}{n}$ converges

$a_n = \frac{\ln(n)}{n}$ $\quad b_n = \frac{1}{n}$

$\frac{\ln(n)}{n} > \frac{1}{n}$

$\sum_{n=3}^{\infty} \frac{1}{n}$ is the harmonic series & diverges

$\therefore \sum_{n=3}^{\infty} \frac{\ln(n)}{n}$ diverges by DCT