February 25

If you use the 5^{th}-degree Maclaurin Polynomial to approximate sin(1) how close are you to the actual answer?
How can you get closer?

$$x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

$$\sin(1) = .84146$$

$$1 - \frac{1}{3!} + \frac{1}{5!} = 0.8416$$

Error = .000195

February 25

Students will verbally explain how to find the error of a Taylor Polynomial using the Lagrange Error Bound

(using the words:
derivative, next term...)

Taylor Polynomial

$$P(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n$$

$$f(x) = P(x) + \underbrace{R(x)}_{\text{error}}$$

$$\sin(1) \approx 1 - \frac{1}{6} + \frac{1}{120} + .000195$$

Lagrange Error Bound

"Remainder in a Taylor Polynomial"

maximum value for the n+1 derivative (for some z between a and x)

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-a)^{n+1}$$

centered at $x=a$

Error

What is the Maximum Error?

$$|R_n(x)| \le M\,\frac{(x-a)^{n+1}}{(n+1)!}$$

Constant

$f^{(n+1)}(z)$

Max value of the derivative

Determine how closely the 4th degree Taylor polynomial for cos(x) approximates the true value on the interval (-0.5, 0.5)

$$|R_4(x)| \le \frac{f^{(5)}(z)}{5!}(x-0)^5$$

$f^{(5)}(z) = 1$

max value for $-\sin x$

$f(x) = \cos x$

$f'(x) = -\sin x$

$f''(x) = -\cos x$

$f'''(x) = \sin x$

$f^{IV}(x) = \cos x$

$f^{(5)}(x) = -\sin x$

$$|R_4(x)| \le \frac{1}{5!}x^5$$

$$|R_4(x)| \le \frac{1}{5!}\left(\frac{1}{2}\right)^5 = \frac{1}{120}\cdot\frac{1}{32}$$

$$|R_4(x)| \le \frac{1}{120(32)} = .00026$$

Determine how closely the 3rd degree Taylor polynomial for e^x approximates the true value on the interval $\underbrace{(1.8, 2.2)}_{\text{centered at } x=2}$

$$|R_3(x)| \le \frac{f^{(4)}(z)}{4!}(x-2)^4$$

$$|R_3(x)| \le \frac{e^{2.2}}{4!}(2.2-2)^4$$

error $\le$ .000602

Let $f(x) = \sin\left(5x + \frac{\pi}{4}\right)$ and let P(x) be the third-degree Taylor Polynomial for f about x = 0. Use the Lagrange error bound to show that $\underbrace{\left|f\left(\frac{1}{10}\right) - P\left(\frac{1}{10}\right)\right|}_{\text{error}} < \frac{1}{100}$

$f(x) = \sin\left(5x + \frac{\pi}{4}\right)$

$f'(x) = 5\cos\left(5x + \frac{\pi}{4}\right)$

$f''(x) = -25\sin\left(5x + \frac{\pi}{4}\right)$

$f'''(x) = -125\cos\left(5x + \frac{\pi}{4}\right)$

$f^{IV}(x) = 625\sin\left(5x + \frac{\pi}{4}\right)$

$$|R_3(x)| \le \frac{f^{IV}(z)}{4!}(x-0)^4$$

$$|R_3(x)| \le \frac{625}{4!}\left(\frac{1}{10}\right)^4$$

$$\frac{625}{24}\left(\frac{1}{10000}\right) < \frac{1}{100}$$