February 3

Find the first four derivatives for the following function:

$$f(x) = \sqrt{2x + 1}$$

$$f'(x) = (2x+1)^{-1/2}$$

$$f''(x) = -\frac{1}{2}(2x+1)^{-\frac{3}{2}}(2) = -(2x+1)^{-3/2}$$

$$f'''(x) = +\frac{3}{2}(2x+1)^{-5/2}(2) = 3(2x+1)^{-5/2}$$

$$f^{IV}(x) = -15(2x+1)^{-7/2}$$

February 3

Students will verbally explain how to represent functions with Taylor Polynomials

(using the words:
substitution, term, derivative, antiderivative..)

Taylor Polynomial Centered at x = 0

$$P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^n(0)}{n!}x^n$$

series

$$f(x) \approx f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n + \cdots$$

Taylor Polynomial Centered at x = a

$$P(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

series

$$f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \cdots$$

Taylor Series/Polynomials

Maclaurin Series

Taylor Polynomial Centered at x = 0

$$P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^n(0)}{n!}x^n$$

$\sin(0) = 0$

$f'(0) = \cos(0) = 1$

$f''(0) = -\sin(0) = 0$

$f'''(0) = -\cos(0) = -1$

$$\sin(x) \approx 0 + 1x + \frac{0}{2!}x^2 + \frac{-1}{3!}x^3$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

Taylor Series/Polynomials

Maclaurin Series

find the 4th degree Taylor Polynomial that approximates $f(x)=\sqrt{1+2x}$ around zero.

$$P(x)=f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\frac{f^{(IV)}(0)}{4!}x^4$$

$$f'(x)=(2x+1)^{-1/2} \longrightarrow f'(0)=1$$

$$f''(x)=\frac{-1}{2}(2x+1)^{-\frac{3}{2}}(2)=-(2x+1)^{-3/2} \rightarrow f''(0)=-1$$

$$f'''(x)=+\frac{3}{2}(2x+1)^{-5/2}(2)=3(2x+1)^{-5/2} \rightarrow f'''(0)=3$$

$$f^{IV}(x)=-15(2x+1)^{-7/2} \longrightarrow f^{(IV)}(0)=-15$$

$$f(0)=1$$

$$P(x)=1+1x+\frac{-1}{2!}x^2+\frac{3}{3!}x^3+\frac{-15}{4!}x^4$$

$$P(x)=1+x-\frac{x^2}{2}+\frac{x^3}{2}-\frac{15}{4!}x^4$$

find the 3rd degree Taylor Polynomial approximation of e^{-x} centered at $x=1$

$$P(x)=f(1)+f'(1)(x-1)+\frac{f''(1)(x-1)^2}{2!}+\frac{f'''(1)(x-1)^3}{3!}$$

$$f(x)=e^{-x} \rightarrow f(1)=e^{-1}=\frac{1}{e}$$

$$f'(x)=-e^{-x} \rightarrow f'(1)=-\frac{1}{e}$$

$$f''(x)=e^{-x} \rightarrow f''(1)=\frac{1}{e}$$

$$f'''(x)=-e^{-x} \rightarrow f'''(1)=-\frac{1}{e}$$

$$P(x)=\frac{1}{e}+\frac{-1}{e}(x-1)+\frac{1(x-1)^2}{e(2!)}+\frac{-1(x-1)^3}{e(3!)}$$

approximate $f(1.1)$ using $P_3(x)$

$$f(1.1)\approx\frac{1}{e}+\frac{-1}{e}(1.1-1)+\frac{1(1.1-1)^2}{e(2!)}+\frac{-1(1.1-1)^3}{e(3!)}$$

$f(x) = 1 + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} + \cdots + \frac{x^n}{(n+1)!} + \cdots$

$f(x)$ is given by the Maclaurin series above

(centered at zero)

find $f'(0)$

find $f^{(10)}(0)$

$f(x) = 1 + \boxed{\frac{x}{2!}} + \frac{x^2}{3!} + \frac{x^3}{4!} + \cdots + \boxed{\frac{x^n}{(n+1)!}} + \cdots$

General Taylor Series

$f(x) = f(0) + \boxed{f'(0)x} + \frac{f''(0)x^2}{2!} + \cdots + \boxed{\frac{f^{(n)}(0)x^n}{n!}} + \cdots$

$\frac{x}{2!} = f'(0)x$

$\frac{1}{2!} = \frac{1}{2} = f'(0)$

$\frac{f^{(10)}(0)x^{10}}{10!} = \frac{x^{10}}{(10+1)!}$

$\frac{f^{(10)}(0)x^{10}}{10!} = \frac{x^{10}}{11!}$

$\frac{f^{(10)}(0)}{10!} = \frac{1}{11!}$

$f^{(10)}(0) = \frac{10!}{11!} = \frac{10\cdot 9\cdot 8\cdot 7\cdots 3\cdot 2\cdot 1}{11\cdot 10\cdot 9\cdot 8\cdot 7\cdots 3\cdot 2\cdot 1} = \frac{1}{11}$