JANUARY 13

Which of the following series may converge? How do you know?

$$\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n \qquad \sum_{n=1}^{\infty}\left(\frac{2n+5}{100n}\right) \qquad \sum_{n=1}^{\infty}\frac{1}{n} \qquad \sum_{n=1}^{\infty}\left(\frac{9}{5}\right)^n$$

$$\lim_{n\to\infty}\left(\frac{3}{4}\right)^n = 0$$

$$\lim_{n\to\infty}\frac{1}{n} = 0$$

$$\lim_{n\to\infty}\frac{2n+5}{100n} = \frac{2}{100} = \frac{1}{50}$$

JANUARY 13

Students will verbally explain how to determine if a geometric series converges and what it converges to

(using the words:
ratio, converge, diverge...)

Geometric Series

Sum of the terms in a geometric sequence

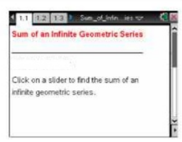
$a + ar + ar^2 + ar^3 + \cdots + ar^n + \cdots$

$$\sum_{n=1}^{\infty} ar^{n-1}$$

**Sum of an Infinite Geometric Series** Name ______________
**Student Activity** Class ______________

**Open the TI-Nspire document *Sum_of_Infinite_Geo_Series.tns*.**

In this activity, you will explore the concept of finding the sum of an infinite geometric series.




**Move to page 1.2.**

Press ctrl ▶ and ctrl ◀ to navigate through the lesson.

1. Press ▲ to see a rectangle with a shaded area of $\frac{1}{2}$ unit$^2$. The length of a side of the original square is 1 unit. What are the dimensions of the rectangle?

2. Press ▲ until the shaded area increases to $\frac{7}{8}$. What are the dimensions of the three rectangles whose sum is $\frac{7}{8}$?

3. What do you expect to be the area of the next rectangle to be added to the sum? (Express your area in both fractional and decimal forms.) Explain how you arrived at your conclusion.

4. Continue pressing ▲ until you can't press it any more. If you could press ▲ again, what would the area of the next rectangle be? What would the total sum of the areas be? (Express your answers in both fractional and decimal forms.)

5. If you could continue pressing ▲ an infinite number of times, and the whole region were shaded, what would the total shaded area be? (Express your answers in both fractional and decimal forms.)

6. Write an expression for the sum of the areas of the infinite number of rectangles formed. What is the value of this sum? Why?

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots+\frac{1}{2^n}=1$

7. Express your answer from Question 6 in sigma notation.

$\sum_{n=1}^{\infty}\frac{1}{2^n}$

8. Instead of halving the side of the square, suppose that we doubled its size and continued to double a side of each subsequent square formed.
   a. Express the sum of the areas of these squares as an infinite sum.

   $2+4+8+\cdots+2^n+\cdots$

   b. What happens to this sum as the number of squares increases? Explain your answer.

9. Instead of halving the length or width of each of the rectangles, suppose that we multiplied the rectangle's length or width by 1/3, giving us the series $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\ldots$

   Do you think that the sum of the series would be finite or infinite? Explain.

10. Give an example of an infinite geometric series that you think would have a finite sum and an example of one that you think would not have a finite sum. Explain your reasoning.

$\sum_{n=1}^{\infty}\left(\frac{1}{200}\right)^n$ $\quad \sum \frac{1}{8^n}$ $\quad \sum_{n=1}^{\infty}\frac{6^n}{31}=\sum\frac{1}{31}(6)^n$

11. Based on the information above, what do you conjecture must be true about the ratio of the consecutive terms of an infinite geometric series for the series to have a finite sum?

**Move to page 1.3.**

12. For what values of the ratio *r* does an infinite geometric series appear to have a finite sum?



| | |
|---|---|
| Geometric Series Converge when | $\lvert r\rvert<1$<br>$\text{Sum}=\frac{a}{1-r}$<br>(a is the first term in the sequence)<br>Converges to $\frac{a}{1-r}$ |
| Interval of Convergence | $-1<r<1$ |
| Diverges when | $\lvert r\rvert\geq 1$ |

$$\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)\left(\frac{5}{4}\right)^{n-1}$$

Diverges

because $r=\frac{5}{4}\geq 1$

$$\sum_{n=1}^{\infty} 7\left(\frac{5}{7}\right)^{n-1}$$

Converges

$r=\frac{5}{7}<1$

Converges to

$$\frac{7}{1-\frac{5}{7}}=\frac{7}{\frac{2}{7}}=\frac{49}{2}$$

$$\sum_{n=2}^{\infty} 3\left(\frac{2}{3}\right)^{n-1}$$

converge

$r=\frac{2}{3}<1$

$$\frac{3\left(\frac{2}{3}\right)^{2-1}}{1-\frac{2}{3}}=\frac{3\left(\frac{2}{3}\right)}{\frac{1}{3}}=\frac{2}{\frac{1}{3}}=6$$