JANUARY 17

How are the following two expressions related?

$$\int_1^{\infty} \left(\frac{1}{x}\right) dx \qquad \sum_{n=1}^{\infty} \frac{1}{n}$$

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Students will verbally explain how to apply the integral test to determine if a series converges or diverges.

(using the words:
function, sequence...)

Integral Test

Let $a_n = f(n)$

where $f(x)$ is positive, decreasing + continuous for $x \geq 1$

If $\int_1^{\infty} f(x)\,dx$ converges,

then $\sum_{n=1}^{\infty} a_n$ converges

If $\int_1^{\infty} f(x)\,dx$ diverges,

then $\sum_{n=1}^{\infty} a_n$ diverges

Does $\sum_{n=1}^{\infty} e^{-\frac{n}{2}}$ converge

$e^{-\frac{n}{2}} = f(n)$

$\int_1^{\infty} e^{-\frac{x}{2}}\,dx$

$= \lim_{b\to\infty} \int_1^{b} e^{-\frac{x}{2}}\,dx = \lim_{b\to\infty} \int_{-\frac{1}{2}}^{-\frac{b}{2}} -2e^{u}\,du$

$u = -\frac{x}{2}$

$du = -\frac{1}{2}dx$

$-2\,du = dx$

$= \lim_{b\to\infty} -2e^{u}\Big|_{-\frac{1}{2}}^{-\frac{b}{2}}$

$= \lim_{b\to\infty} -2e^{-\frac{b}{2}} + 2e^{-1/2}$

$= 0 + 2e^{-1/2} = 2e^{-1/2}$

$\therefore \int_1^{\infty} e^{-\frac{x}{2}}\,dx$ converges

so $\sum_{n=1}^{\infty} e^{-\frac{n}{2}}$ converges by the integral test

Does the series:

$$\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$$

converge

$a_n = \frac{\ln(n)}{n} = f(n)$

$$\int_1^{\infty} \frac{\ln(x)}{x}\,dx = \lim_{b\to\infty} \int_1^b \frac{\ln(x)}{x}\,dx$$

$u = \ln(x)$

$du = \frac{1}{x}\,dx$

$x \cdot du = dx$

$$= \lim_{b\to\infty} \int_0^{\ln(b)} \frac{u}{x} \cdot x \cdot du$$

$$= \lim_{b\to\infty} \int_0^{\ln(b)} u\,du$$

$$= \lim_{b\to\infty} \frac{u^2}{2}\Bigg|_0^{\ln(b)}$$

$$= \lim_{b\to\infty} \frac{\ln(b)^2}{2} - \frac{0^2}{2} \Rightarrow \infty$$

$\int_1^{\infty} \frac{\ln x}{x}\,dx$ diverges,

so $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$ diverges

by the integral test