JANUARY 21

How can integrals help you determine if a series converges?

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Students will verbally explain how to determine if an improper integral converges using the comparison test

(using the words:
converge, diverge, greater, less...)

Name: ____

Complete the table below

Series	Type of Series (Geometric, P-series, neither)	Converge, Diverge, or possibly converge	If it converges, what does it converge to?
$\sum_{n=1}^{\infty} 6\left(\frac{4}{5}\right)^{n-1}$	Geometric	converge	30
$\sum_{n=1}^{\infty} \frac{1}{n^7}$	P-series	converge	
$\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^5}}$	P-series	converge	
$\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)\left(\frac{4}{3}\right)^{n-1}$	Geometric	diverge	
$\sum_{n=1}^{\infty} \frac{n+1}{3n+1}$	Neither	diverge	
$\sum_{n=1}^{\infty} \left(\frac{\pi}{6}\right)^{n-1}$	Geometric	converge	6/(6-π)
$\sum_{n=1}^{\infty} \frac{5}{3n+1}$	Neither	may converge	
$\sum_{n=1}^{\infty} \frac{1}{n^{-3}}$	P-series	diverge	
$\sum_{n=1}^{\infty} (-1)^n \frac{3}{4n+1}$	Neither	may converge	
$\sum_{n=1}^{\infty} \left(\frac{\pi}{e}\right)^{n-1}$	Geometric	diverge	

Question 4

Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \frac{x^2 - 2}{x}$ for all $x \neq 0$.

$h'(x) = \frac{2x(x) - (x^2 - 2)}{x^2} = \frac{x^2 + 2}{x^2}$

(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

(c) Write an equation for the line tangent to the graph of h at $x = 4$.

(d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

(a) $h'(x) = 0$ at $x = \pm\sqrt{2}$

x		$-\sqrt{2}$		0		$\sqrt{2}$	
$h'(x)$	$-$	0	$+$	und	$-$	0	$+$

Local minima at $x = -\sqrt{2}$ and at $x = \sqrt{2}$

because $h'(x)$ changes from $-$ to $+$

4 : { 1 : $x = \pm\sqrt{2}$
1 : analysis
2 : conclusions
< -1 > not dealing with discontinuity at 0 }

CP	$-\sqrt{2}$		0		$\sqrt{2}$	
sign h'	$-$		$+$		$-$	$+$
behav h						

(b) $h''(x) = 1 + \frac{2}{x^2} > 0$ for all $x \neq 0$. Therefore, the graph of h is concave up for all $x \neq 0$.

3 : { 1 : $h''(x)$
1 : $h''(x) > 0$
1 : answer }

(c) $h'(4) = \frac{16 - 2}{4} = \frac{7}{2}$

$y + 3 = \frac{7}{2}(x - 4)$

1 : tangent line equation

(d) The tangent line is below the graph because the graph of h is concave up for $x > 4$.

1 : answer with reason

Question 5

Let f be the function satisfying $f'(x) = -3xf(x)$, for all real numbers x, with $f(1) = 4$ and $\lim_{x\to\infty} f(x) = 0$.

(a) Evaluate $\int_1^{\infty} -3xf(x)\,dx$. Show the work that leads to your answer.

(b) Use Euler's method, starting at $x = 1$ with a step size of 0.5, to approximate $f(2)$.

(c) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = -3xy$ with the initial condition $f(1) = 4$.

(a) $\int_1^{\infty} -3xf(x)\,dx$

$= \int_1^{\infty} f'(x)\,dx = \lim_{b\to\infty}\int_1^{b} f'(x)\,dx = \lim_{b\to\infty} f(x)\Big|_1^b$

$= \lim_{b\to\infty} f(b) - f(1) = 0 - 4 = -4$

$f(\infty) - f(1)$

2 : $\begin{cases} 1 : \text{use of FTC} \\ 1 : \text{answer from limiting process} \end{cases}$

(b) $f(1.5) \approx f(1) + f'(1)(0.5)$

$= 4 - 3(1)(4)(0.5) = -2$

$f(2) \approx -2 + f'(1.5)(0.5)$

$\approx -2 - 3(1.5)(-2)(0.5) = 2.5$

$f(2) \approx 2.5$

2 : $\begin{cases} 1 : \text{Euler's method equations or equivalent table} \\ 1 : \text{Euler approximation to } f(2) \\ \quad \text{(not eligible without first point)} \end{cases}$

(c) $\int \frac{1}{y}\,dy = \int -3x\,dx$

$\ln y = -\frac{3}{2}x^2 + k$

$-\frac{3}{2}x^2 + c$

$y = e$

$= e^{-\frac{3}{2}x^2} e^c$

$y = Ce^{-\frac{3}{2}x^2}$

$4 = Ce^{-\frac{3}{2}}$; $C = 4e^{\frac{3}{2}}$

$y = 4e^{\frac{3}{2}}e^{-\frac{3}{2}x^2}$

$4e^{3/2}e^{-3/2x^2}$

$4 = e^{-\frac{3}{2}(1)^2 + c}$

5 : $\begin{cases} 1 : \text{separates variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition } f(1) = 4 \\ 1 : \text{solves for } y \end{cases}$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables