January 24

In a well written paragraph, describe what you know about series and why they can be useful.

January 24

Students will verbally explain how to express rational functions as a power series

(using the words:
error, approximate, series...)

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\cos x \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots + (-1)^n \frac{x^{2n}}{(2n)!} + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots + \frac{x^n}{n!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Express the following series in sigma notation

$1 + .1 + .01 + .001 + \ldots$

$$\boxed{\cancel{\sum_{n=0}^{\infty} 1^{-n} = \sum_{n=0}^{\infty} \frac{1}{1^n}}}$$

$$\sum_{n=0}^{\infty} \frac{1}{10^n} = 1 + \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \cdots$$

$$1 + .1 + .01 + .001$$

Converge because $r = \frac{1}{10} < 1$

(geometric series)

Converge to $\frac{1}{1 - \frac{1}{10}} = \frac{10}{9}$

Geometric Series:

Geometric series are represented by $\sum_{n=1}^{\infty} a(r)^{n-1}$.

A geometric series converges when $|r|<1 \rightarrow -1<r<1$

The sum of a geometric series is $\dfrac{a}{1-r}$

Generalize the formula by substituting 1 for *S* and x for *r*. Write the first 5 terms of the expansion when *a* = 1 and *r* = *x*.

$$\frac{1}{1-x} = \sum_{n=1}^{\infty} 1(x)^{n-1} = 1 + x + x^2 + x^3 + x^4 + \cdots$$ ← equation 2 in book

$a=1 \quad r=x$

What is the **interval of convergence** for $f(x)=\dfrac{1}{1-x}$?

$$|x|<1 \rightarrow -1<x<1$$

Interval of Convergence (I.C)

x-values where the series "match" with the function

$$\frac{1}{1-x} = \sum_{n=1}^{\infty} x^{n-1} \qquad \text{IoC: } -1<x<1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad \text{IoC: } -\infty<x<\infty$$

Rational Functions written as a series of polynomials are called **Power Series**.

Power Series

An expression in the form

$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + \ldots + c_n x^n + \ldots$$

Is a **power series centered at x = 0**.

An expression in the form

$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \ldots + c_n(x-a)^n + \ldots$$

Is a **power series centered at x = a**

The power series we just developed can be used a template to find power series of other functions.

Given that $\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots + x^n + \ldots = \sum_{n=0}^{\infty} x^n$, do the following examples.

Example 7. Write a power series that in both expanded form and sigma notation that represents $\dfrac{1}{1+x}$. What is the interval of convergence?

$$\frac{1}{1+x} = \frac{1}{1-(-x)} = 1 + (-x) + (-x)^2 + (-x)^3 + \cdots + (-x)^n + \cdots$$

$$= 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots + (-1)^n x^n + \cdots$$

$$= \sum_{n=0}^{\infty} (-1)^n x^n = \sum_{n=0}^{\infty} (-x)^n$$

IoC

$|-x| < 1$

$-1 < -x < 1$

$-1 < x < 1$

The power series we just developed can be used a template to find power series of other functions.

Given that $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots + x^n + \ldots = \sum_{n=0}^{\infty} x^n$, do the following examples.

Example 8. Write a power series that in both expanded form and sigma notation that represents $\frac{x}{1+x}$. What is the interval of convergence?

$$\frac{x}{1+x} = \frac{x}{1-(-x)} = x\left(\frac{1}{1-(-x)}\right)$$

$$= x\left(1+(-x)+(-x)^2+(-x)^3+\cdots+(-x)^n\right)$$

$$= x\left(1-x+x^2-x^3+\cdots+(-1)^n x^n\right)$$

$$= x - x^2 + x^3 - x^4 + \cdots + (-1)^n x^{n+1} + \cdots = \sum_{n=0}^{\infty} (-1)^n x^{n+1}$$

$$= \sum_{\infty}^{n=0} (-x)^n (x)$$

IoC

$-1 < x < 1$

The power series we just developed can be used a template to find power series of other functions.

Given that $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots + x^n + \ldots = \sum_{n=0}^{\infty} x^n$, do the following examples.

Example 9. Write a power series that in both expanded form and sigma notation that represents $\frac{1}{1-3x}$. What is the interval of convergence?

$$\frac{1}{1-(3x)} = 1+(3x)+(3x)^2+(3x)^3+\cdots+(3x)^n+\cdots$$

$$1+(3x)+(3x)^2+(3x)^3+\cdots+(3x)^n+\cdots$$

$$= \sum_{n=0}^{\infty} (3x)^n = \sum_{n=0}^{\infty} 3^n x^n$$

IoC

$|3x| < 1$

$$\frac{-1}{3} < \frac{3x}{3} < \frac{1}{3}$$

$-\frac{1}{3} < x < \frac{1}{3}$