January 27

How can the power series for

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots$$

be used to create series for other functions?

$$\frac{1}{1+x} = \frac{1}{1-(-x)} = 1 + (-x) + (-x)^2 + \cdots + (-x)^n + \cdots$$

January 27

Students will verbally explain how to use differentiation and integration to find power series

(using the words:
substitution, term, derivative, antiderivative..)

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots + x^n + \ldots = \sum_{n=0}^{\infty} x^n$$

$$\cos x \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots + (-1)^n \frac{x^{2n}}{(2n)!} + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots + \frac{x^n}{n!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

find the power series expansion for $\frac{1}{(1-x)^2}$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots = \sum_{n=0}^{\infty} x^n$$

$$\frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{d}{dx}\left(1 + x + x^2 + x^3 + \cdots + x^n + \cdots\right)$$

$$\frac{1}{(1-x)^2} = 0 + 1 + 2x + 3x^2 + \cdots + nx^{n-1} + \cdots = \sum_{n=0}^{\infty} nx^{n-1}$$

IoC: $-1 < x < 1$

Evaluate $\sum_{n=1}^{\infty} \frac{n}{3^n}$

$$= \sum_{n=1}^{\infty} n\left(\frac{1}{3^n}\right) = \sum_{n=1}^{\infty} n\left(\frac{1}{3}\right)^n = \sum_{n=1}^{\infty} n\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^{n-1} = \frac{1}{3}\sum_{n=1}^{\infty} n\left(\frac{1}{3}\right)^{n-1}$$

$x = \frac{1}{3}$

$$= \frac{1}{3} \cdot \frac{1}{(1-x)^2} = \frac{1}{3}\left(\frac{1}{\left(1-\frac{1}{3}\right)^2}\right) = \frac{3}{4}$$

find the power series expansion for $\frac{1}{1+x^2}$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots = \sum_{n=0}^{\infty} x^n$$

$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = 1 + (-x^2) + (-x^2)^2 + (-x^2)^3 + \cdots + (-x^2)^n + \cdots$$

$$= 1 - x^2 + x^4 - x^6 + \cdots + (-x^2)^n + \cdots = \sum_{n=0}^{\infty} (-1)^n x^{2n}$$

$$+ (-1)^n x^{2n} + \cdots$$

IoC: $-1 < x^2 < 1$ $\rightarrow |x| < 1$

$\cancel{\sqrt{-1}} < x < \pm\sqrt{1} \rightarrow -1 < x < 1$

find the power series expansion for $\tan^{-1} x$

$$\int \frac{1}{1+x^2}\,dx = \int 1 - x^2 + x^4 - x^6 + \cdots + (-1)^n x^{2n} + \cdots\, dx$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

IoC: $-1 < x < 1$

find the power series expansion for $\ln(1+x)$

(using $\frac{1}{1+x} = \int 1 - x + x^2 - x^3 + x^4 + \cdots$)

$$\int \frac{1}{1+x}\,dx = \ln(1+x)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1} \frac{x^n}{n} + \cdots$$