January 28

What is an interval of convergence?
How does it change when you take the derivative or integral?

January 28

Students will verbally explain how to find the power series representation of a given function

(using the words:
substitution, term, derivative, antiderivative..)

???

???

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + (-1)^n\left(\frac{x^{2n+1}}{(2n+1)!}\right) + \cdots = \sum_{n=0}^{\infty}(-1)^n\left(\frac{x^{2n+1}}{(2n+1)!}\right)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + (-1)^n\left(\frac{x^{2n}}{(2n)!}\right) + \cdots = \sum_{n=0}^{\infty}(-1)^n\left(\frac{x^{2n}}{(2n)!}\right)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots = \sum_{n=0}^{\infty}\frac{x^n}{n!}$$

IoC $-\infty < x < \infty$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots = \sum_{n=0}^{\infty} x^n$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots + (-1)^n x^n + \cdots = \sum_{n=0}^{\infty}(-1)^n x^n$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n-1}\frac{x^n}{n} + \cdots = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}$$

$$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots + (-1)^n\frac{x^{2n+1}}{2n+1} + \cdots = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}$$

$-1 < x < 1$

Maclaurin Series

Find the Maclaurin series for $f(x) = \sin(2x)$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots$$

$$\sin(2x) = (2x) - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \frac{(2x)^7}{7!} + \cdots + (-1)^n \frac{(2x)^{2n+1}}{(2n+1)!} + \cdots$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{2n+1}}{(2n+1)!}$$

Write the first 3 nonzero terms and the general term for the Maclaurin series for

$f(x) = \ln\left(1 + 4x^2\right)$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n-1}\frac{x^n}{n} + \cdots$$

$$\ln(1+4x^2) = (4x^2) - \frac{(4x^2)^2}{2} + \frac{(4x^2)^3}{3} - \frac{(4x^2)^4}{4} + \cdots + (-1)^{n-1}\frac{(4x^2)^n}{n} + \cdots$$

$$\ln(1+4x^2) = 4x^2 - \frac{4^2x^4}{2} + \frac{4^3x^6}{3} + \cdots + \frac{(-1)^{n-1}(4)^n x^{2n}}{n} + \cdots$$

$f(x) = 3x\tan^{-1}\left(x^2\right)$

first 3 nonzero + general

$$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} + \cdots + (-1)^n\frac{x^{2n+1}}{2n+1} + \cdots$$

$$\tan^{-1}(x^2) = (x^2) - \frac{(x^2)^3}{3} + \frac{(x^2)^5}{5} + \cdots + (-1)^n\frac{(x^2)^{2n+1}}{2n+1} + \cdots$$

$$= x^2 - \frac{x^6}{3} + \frac{x^{10}}{5} + \cdots + \frac{(-1)^n x^{4n+2}}{2n+1} + \cdots$$

$$3x\tan^{-1}(x^2) = 3x\left(x^2 - \frac{x^6}{3} + \frac{x^{10}}{5} + \cdots + \frac{(-1)^n x^{4n+2}}{2n+1} + \cdots\right)$$

$$= 3x^3 - \frac{3x^7}{3} + \frac{3x^{11}}{5} + \cdots + \frac{(-1)^n 3x^1(x)^{4n+2}}{2n+1} + \cdots$$

$$= 3x^3 - x^7 + \frac{3x^{11}}{5} + \cdots + \frac{(-1)^n(3)(x)^{4n+3}}{2n+1} + \cdots$$