March 10

How is arc length for a parametric function different than arc length for a "regular" function?

March 7

Students will verbally explain how to use vectors to describe position, velocity and acceleration

(using the words: derivatives, integrals, magnitude...)

CALCULUS SESSION WED.
12:00 - 2:00

1. An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \ge 0$ with

$$\frac{dx}{dt} = 12t - 3t^2 \text{ and } \frac{dy}{dt} = \ln\left(1 + (t-4)^4\right).$$

At time $t = 0$, the object is at position $(-13, 5)$. At time $t = 2$, the object is at point P with x-coordinate 3.

(a) Find the acceleration vector at time $t = 2$ and the speed at time $t = 2$.

(b) Find the y-coordinate of P.

(c) Write an equation for the line tangent to the curve at P.

(d) For what value of t, if any, is the object at rest? Explain your reasoning.

$\int_0^2 \frac{dy}{dt}\, dt = y(2) - y(0)$

$8.6714 = y(2) - 5$

$13.6714 = y(2)$

WRITE ALL WORK IN THE TEST BOOKLET.

$\text{Slope} = \frac{y'(2)}{x'(2)} = .236$

$y - 13.671 = .236(x-3)$

$12t - 3t^2 = 0$

$3t(4-t) = 0$

$t = 0, 4$

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AP® CALCULUS BC
2005 SCORING GUIDELINES (Form B)

Question 1

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(c) Write an equation for the line tangent to the curve at P.

(d) For what value of t, if any, is the object at rest? Explain your reasoning.

(a) $x''(2) = 0,\ y''(2) = -\frac{32}{17} = -1.882$ $a(2) = \langle 0, -1.882 \rangle$ Speed $= \sqrt{12^2 + (\ln(17))^2} = 12.329$ or 12.330	$2: \begin{cases} 1: \text{acceleration vector} \\ 1: \text{speed} \end{cases}$
(b) $y(t) = y(0) + \int_0^t \ln\left(1 + (u-4)^4\right) du$ $y(2) = 5 + \int_0^2 \ln\left(1 + (u-4)^4\right) du = 13.671$	$3: \begin{cases} 1: \int_0^2 \ln\left(1 + (u-4)^4\right) du \\ 1: \text{handles initial condition} \\ 1: \text{answer} \end{cases}$
(c) At $t = 2$, slope $= \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{\ln(17)}{12} = 0.236$ $y - 13.671 = 0.236(x-3)$	$2: \begin{cases} 1: \text{slope} \\ 1: \text{equation} \end{cases}$
(d) $x'(t) = 0$ if $t = 0, 4$ $y'(t) = 0$ if $t = 4$ $t = 4$	$2: \begin{cases} 1: \text{reason} \\ 1: \text{answer} \end{cases}$

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. A particle moving along a curve in the plane has position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \sqrt{t^4 + 9} \text{ and } \frac{dy}{dt} = 2e^t + 5e^{-t}$$

for all real values of t. At time $t = 0$, the particle is at the point $(4, 1)$.

(a) Find the speed of the particle and its acceleration vector at time $t = 0$.

(b) Find an equation of the line tangent to the path of the particle at time $t = 0$.

(c) Find the total distance traveled by the particle over the time interval $0 \le t \le 3$.

(d) Find the x-coordinate of the position of the particle at time $t = 3$.

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2. Let f be a function having derivatives of all orders for all real numbers. The third-degree Taylor polynomial for f about $x = 2$ is given by

$$T(x) = 7 - 9(x-2)^2 - 3(x-2)^3.$$

(a) Find $f(2)$ and $f''(2)$.

(b) Is there enough information given to determine whether f has a critical point at $x = 2$?

If not, explain why not.

If so, determine whether $f(2)$ is a relative maximum, a relative minimum, or neither, and justify your answer.

(c) Use $T(x)$ to find an approximation for $f(0)$. Is there enough information given to determine whether f has a critical point at $x = 0$?

If not, explain why not.

If so, determine whether $f(0)$ is a relative maximum, a relative minimum, or neither, and justify your answer.

(d) The fourth derivative of f satisfies the inequality $\left|f^{(4)}(x)\right| \le 6$ for all x in the closed interval $[0, 2]$. Use the Lagrange error bound on the approximation to $f(0)$ found in part (c) to explain why $f(0)$ is negative.

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Question 1

A particle moving along a curve in the plane has position $(x(t), y(t))$ at time t, where

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(a) Find the speed of the particle and its acceleration vector at time $t = 0$.
(b) Find an equation of the line tangent to the path of the particle at time $t = 0$.
(c) Find the total distance traveled by the particle over the time interval $0 \le t \le 3$.
(d) Find the x-coordinate of the position of the particle at time $t = 3$.

(a) At time $t = 0$: Speed $= \sqrt{x'(0)^2 + y'(0)^2} = \sqrt{3^2 + 7^2} = \sqrt{58}$ Acceleration vector $= \langle x''(0), y''(0)\rangle = \langle 0, -3\rangle$	$2: \begin{cases} 1: \text{speed} \\ 1: \text{acceleration vector} \end{cases}$
(b) $\dfrac{dy}{dx} = \dfrac{y'(0)}{x'(0)} = \dfrac{7}{3}$ Tangent line is $y = \frac{7}{3}(x-4) + 1$	$2: \begin{cases} 1: \text{slope} \\ 1: \text{tangent line} \end{cases}$
(c) Distance $= \int_0^3 \sqrt{\left(\sqrt{t^4+9}\right)^2 + \left(2e^t + 5e^{-t}\right)^2}\, dt$ $= 45.226$ or 45.227	$3: \begin{cases} 2: \text{distance integral} \\ \quad \langle -1 \rangle \text{ each integrand error} \\ \quad \langle -1 \rangle \text{ error in limits} \\ 1: \text{answer} \end{cases}$
(d) $x(3) = 4 + \int_0^3 \sqrt{t^4 + 9}\, dt$ $= 17.930$ or 17.931	$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

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2004 SCORING GUIDELINES (Form B)

Question 2

Let f be a function having derivatives of all orders for all real numbers. The third-degree Taylor polynomial for f about $x = 2$ is given by $T(x) = 7 - 9(x-2)^2 - 3(x-2)^3$.

(a) Find $f(2)$ and $f''(2)$.

(b) Is there enough information given to determine whether f has a critical point at $x = 2$? If not, explain why not. If so, determine whether $f(2)$ is a relative maximum, a relative minimum, or neither, and justify your answer.

(c) Use $T(x)$ to find an approximation for $f(0)$. Is there enough information given to determine whether f has a critical point at $x = 0$? If not, explain why not. If so, determine whether $f(0)$ is a relative maximum, a relative minimum, or neither, and justify your answer.

(d) The fourth derivative of f satisfies the inequality $\left|f^{(4)}(x)\right| \le 6$ for all x in the closed interval $[0, 2]$. Use the Lagrange error bound on the approximation to $f(0)$ found in part (c) to explain why $f(0)$ is negative.

(a) $f(2) = T(2) = 7$ $\frac{f''(2)}{2!} = -9$ so $f''(2) = -18$	$2: \begin{cases} 1: f(2) = 7 \\ 1: f''(2) = -18 \end{cases}$
(b) Yes, since $f'(2) = T'(2) = 0$, f does have a critical point at $x = 2$. Since $f''(2) = -18 < 0$, $f(2)$ is a relative maximum value.	$2: \begin{cases} 1: \text{states } f'(2) = 0 \\ 1: \text{declares } f(2) \text{ as a relative} \\ \quad \text{maximum because } f''(2) < 0 \end{cases}$
(c) $f(0) \approx T(0) = -5$ It is not possible to determine if f has a critical point at $x = 0$ because $T(x)$ gives exact information only at $x = 2$.	$3: \begin{cases} 1: f(0) \approx T(0) = -5 \\ 1: \text{declares that it is not} \\ \quad \text{possible to determine} \\ 1: \text{reason} \end{cases}$
(d) Lagrange error bound $= \frac{6}{4!}\lvert 0-2\rvert^4 = 4$ $f(0) \le T(0) + 4 = -1$ Therefore, $f(0)$ is negative.	$2: \begin{cases} 1: \text{value of Lagrange error} \\ \quad \text{bound} \\ 1: \text{explanation} \end{cases}$