March 18

Mock Exam:
March 27th - 7:30 am
Room 5

How are polar coordinates/functions different from rectangular coordinates/functions?

March 18

Students will verbally explain how to graph polar functions and find their derivatives

(using the words:
radius, angle, polar...)

b. Describe the location of the point with the following polar coordinates:

(i) $r > 0$ and $\theta = 0$

(ii) $r < 0$ and $\theta = \frac{3\pi}{2}$

(iii) $r < 0$ and $\theta = \frac{\pi}{2}$

(iv) $r > 0$ and $\theta = -3\pi$

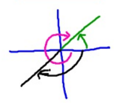
Move to page 2.1.

If a point has polar coordinates (r,θ), then the rectangular coordinates are given by $\boxed{x = r\cos\theta}$ and $\boxed{y = r\sin\theta}$. Similarly, if a point has rectangular coordinates (x, y), then the polar coordinates are (r,θ) such that $\boxed{r^2 = x^2 + y^2}$ and $\boxed{\tan\theta = \frac{y}{x}}$, $x \neq 0$.

2. Complete each of the following tables. Use Page 2.1 to enter polar coordinates and/or rectangular coordinates, to plot the points, and to check your answers. Enter coordinates in the left work area in the appropriate Math Box. The polar coordinates are represented by the point P and the rectangular coordinates are represented by the point R.

a. Find two additional ways to represent each given rectangular point in polar coordinates.

(r_1,θ_1)	$\left(2,\frac{\pi}{4}\right)$	$\left(3,\frac{7\pi}{4}\right)$	$\left(6,\frac{2\pi}{3}\right)$	$\left(1,\frac{7\pi}{6}\right)$	$\left(-2,\frac{5\pi}{4}\right)$	$\left(\frac{3}{4},\frac{17\pi}{6}\right)$
(r_2,θ_2)	$\left(2,-\frac{7\pi}{4}\right)$			$\left(-1,\frac{\pi}{6}\right)$		
(r_3,θ_3)	$\left(-2,-\frac{3\pi}{4}\right)$			$\left(1,-\frac{5\pi}{6}\right)$		



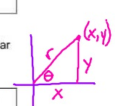
$x = r\cos\theta$ $y = r\sin\theta$

b. For each point given in polar coordinates below, determine the rectangular coordinates.

(r,θ)	$\left(3,\frac{7\pi}{3}\right)$	$\left(1,\frac{\pi}{6}\right)$	$\left(-2,-\frac{4\pi}{3}\right)$	$\left(\sqrt{5},-\frac{3\pi}{2}\right)$	$\left(-8,\frac{3\pi}{4}\right)$	$\left(\frac{13}{4},-\frac{\pi}{3}\right)$
x	$3\cos\left(\frac{7\pi}{3}\right) = 3\left(\frac{1}{2}\right) = \frac{3}{2}$			$\sqrt{5}\cos\left(-\frac{3\pi}{2}\right) = 0$		
y	$3\sin\left(\frac{7\pi}{3}\right) = 3\left(\frac{\sqrt{3}}{2}\right) = \frac{3\sqrt{3}}{2}$			$\sqrt{5}\sin\left(-\frac{3\pi}{2}\right) = \sqrt{5}$		

c. For each point given in rectangular coordinates below, determine two representations in polar coordinates.

$r^2 = x^2 + y^2$ $\tan\theta = \frac{y}{x}$



(x, y)	$(3,4)$	$(-\sqrt{2},2)$	$(4,-7)$	$(-\sqrt{3},-1)$	$(-5,5)$	$(7,24)$
r_1	5					
θ_1	$\tan^{-1}\left(\frac{4}{3}\right) = .927$					
r_2	-5					
θ_2	$.927+\pi = 4.069$					

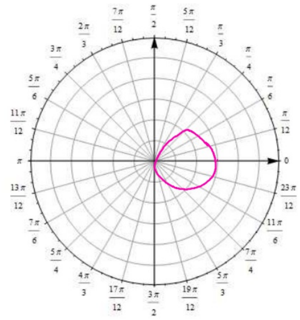
Move to page 3.2.

3. Over the next two worksheet pages, carefully sketch a complete graph of each given polar equation. Create a table of values, search for patterns, and sketch the graph on the axes provided. Check you results on the handheld, and sketch the graph in the right work area of Page 3.2.

Note: Make sure the Graph Type is set to Polar. Use the clicker in the left panel to step through specific points on the curve in polar and rectangular form.

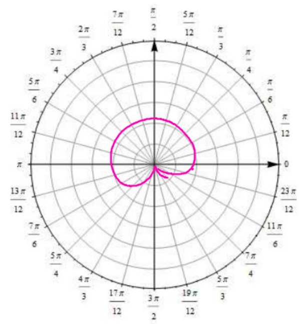
a. $r = 4\cos\theta$.

θ	r



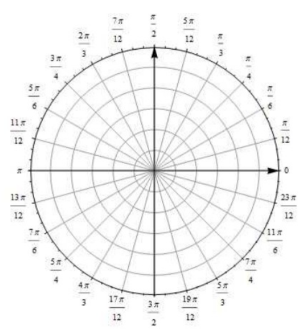
b. $r = 2 + 2\sin\theta$.

θ	r



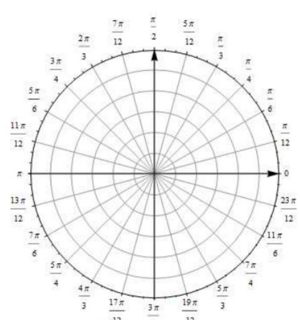
c. $r = 4\cos 3\theta$.

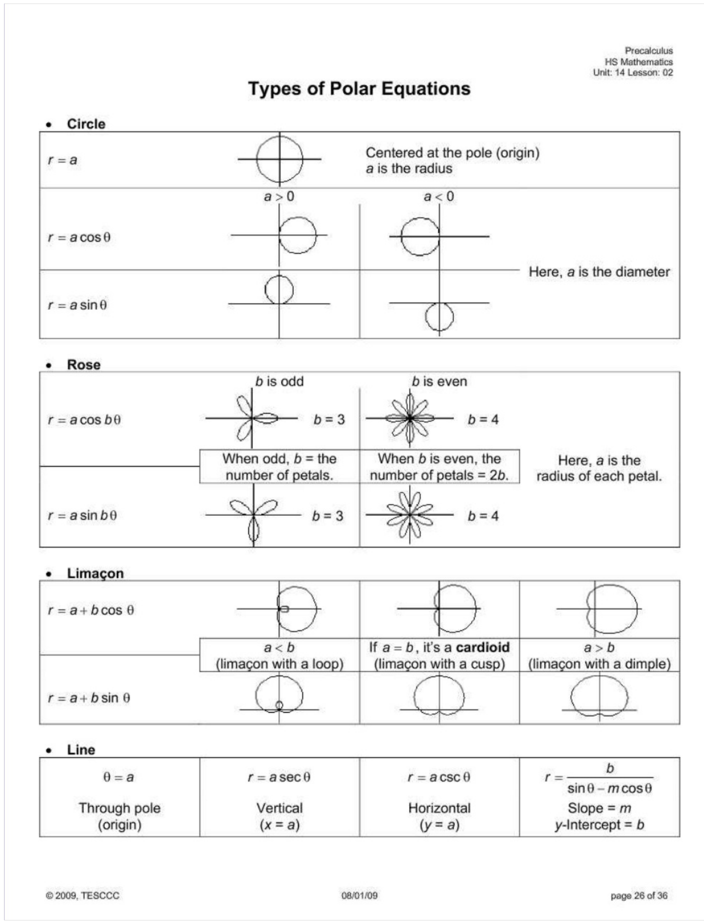
θ	r



d. $r = 1 + 2\cos\theta$.

θ	r





Types of Polar Equations

- **Circle**

$r = a$		Centered at the pole (origin) a is the radius	
	$a > 0$	$a < 0$	
$r = a\cos\theta$			Here, a is the diameter
$r = a\sin\theta$			

- **Rose**

	b is odd	b is even	
$r = a\cos b\theta$	$b = 3$	$b = 4$	
	When odd, b = the number of petals.	When b is even, the number of petals = $2b$.	Here, a is the radius of each petal.
$r = a\sin b\theta$	$b = 3$	$b = 4$	

- **Limaçon**

$r = a + b\cos\theta$			
	$a < b$ (limaçon with a loop)	If $a = b$, it's a **cardioid** (limaçon with a cusp)	$a > b$ (limaçon with a dimple)
$r = a + b\sin\theta$			

- **Line**

$\theta = a$	$r = a\sec\theta$	$r = a\csc\theta$	$r = \dfrac{b}{\sin\theta - m\cos\theta}$
Through pole (origin)	Vertical ($x = a$)	Horizontal ($y = a$)	Slope = m y-Intercept = b

What does $\frac{dr}{d\theta}$ mean?

derivative of r in terms of θ $\rightarrow$ how fast the radius is changing with respect to the angle

What does the slope of a line represent?

$\frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$

How can we find $\frac{dy}{dx}$ with polar functions?

① convert to $x + y$

$x = r\cos\theta$

$y = r\sin\theta$

② find $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$

③ divide $\dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

find the equation of the line tangent to $r=\cos 2\theta$ at $\theta=\frac{\pi}{2}$

$x=r\cos\theta$

$x=\cos 2\theta\cos\theta$ $\qquad x=\cos\left(2\cdot\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right)=0$

$y=r\sin\theta$

$y=\cos 2\theta\sin\theta$ $\qquad y=\cos\left(2\cdot\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right)=-1$

$$\frac{\frac{dy}{d\theta}=-2\sin 2\theta\sin\theta+\cos\theta\cos 2\theta}{\frac{dx}{d\theta}=-2\sin 2\theta\cos\theta-\sin\theta\cos 2\theta}=\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{-2\sin\left(2\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right)+\cos\left(\frac{\pi}{2}\right)\cos\left(2\cdot\frac{\pi}{2}\right)}{-2\sin\left(2\cdot\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right)-\sin\left(\frac{\pi}{2}\right)\cos\left(2\cdot\frac{\pi}{2}\right)}=\frac{-2(0)(1)+0(-1)}{-2(0)(0)-1(-1)}=\frac{0}{1}=0$$

slope

$$y+1=0(x-0)$$

$$y=-1$$