March 20

Given a polar curve, how can you find an equation of a tangent line?

$x = r\cos\theta$

$y = r\sin\theta$

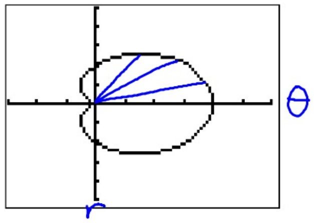
$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \text{slope}$$

March 20

Students will verbally explain how to find the area enclosed by polar curves

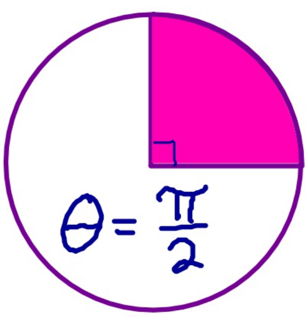
(using the words:
radius, angle, arc...)

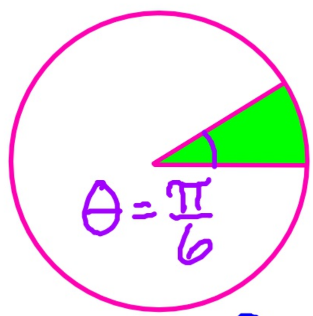
Find the area enclosed by $r = 2(1 + \cos\theta)$



$$\int r \, d\theta$$

Let's simplify it and look at a circle:





$$A = \frac{\pi r^2}{4} \qquad A = \frac{\pi r^2}{12}$$

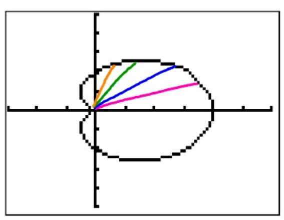
$$A = \frac{1}{2} \cdot \frac{\pi}{2} r^2 \qquad A = \frac{1}{2} \cdot \frac{\pi}{6} r^2$$

Area of sector

$$A = \frac{1}{2}\theta r^2 = \frac{1}{2} r^2 \theta$$

↑ in radians

Find the area enclosed by
$r = 2(1 + \cos \theta)$



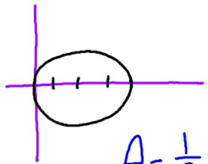
Area of sectors
w/ θ as small
as possible ($d\theta$)

$$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta = \frac{1}{2}\int_{0}^{2\pi} \left(2(1+\cos\theta)\right)^2 d\theta = 18.849$$

Area of Polar Curves

$$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta$$

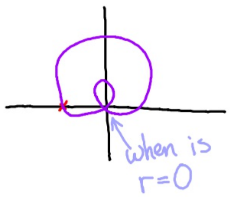
find the area enclosed by $r = 4\cos\theta$



$$A = \frac{1}{2}\int_0^{\pi} (4\cos\theta)^2 \, d\theta$$

$= 12.566$

find the area inside the smaller loop of $r = 2\sin\theta - 1$



$0 = 2\sin\theta - 1$

$1 = 2\sin\theta$

$\frac{1}{2} = \sin\theta$

$\frac{\pi}{6}, \frac{5\pi}{6} = \theta$

$$A = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (2\sin\theta - 1)^2 \, d\theta$$

$= .544$

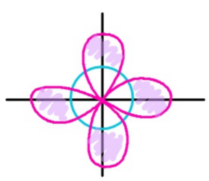
Area between curves

$$A = \frac{1}{2}\int_{\alpha}^{\beta} (r_2)^2 - (r_1)^2 \, d\theta$$

r_2: outside radius

r_1: inside radius

find the area inside $r = 4\cos 2\theta$ & outside $r = 2$



$4\cos 2\theta = 2$

$\cos 2\theta = \frac{1}{2}$

$2\theta = \frac{\pi}{3}, \frac{5\pi}{3}, -\frac{\pi}{3}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, -\frac{\pi}{6}$

$$A_{\text{one petal}} = \frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (4\cos 2\theta)^2 - 2^2 \, d\theta$$

$$A_{\text{total}} = 4\left(\frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (4\cos 2\theta)^2 - 2^2 \, d\theta\right)$$

$= 15.306$