March 24

Mock Exam:
March 27th - 7:30 am
Room 5

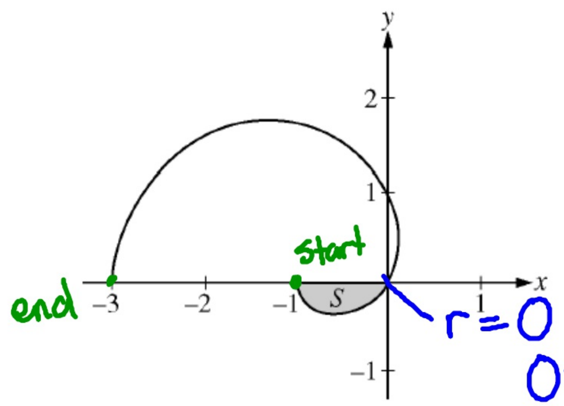
How are parametric functions similar to polar functions?

March 24

Students will verbally explain how to find the area enclosed by polar curves

(using the words:
radius, angle, arc...)

No calculator is allowed for these problems.



$0=1-2\cos\theta$

4. The graph of the polar curve $r = 1 - 2\cos\theta$ for $0 \le \theta \le \pi$ is shown above. Let S be the shaded region in the third quadrant bounded by the curve and the x-axis.

   (a) Write an integral expression for the area of S. $\frac{1}{2}\int_0 r^2\,d\theta$

   (b) Write expressions for $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ in terms of θ. $x=r\cos\theta$, $y=r\sin\theta$

   (c) Write an equation in terms of x and y for the line tangent to the graph of the polar curve at the point where $\theta = \frac{\pi}{2}$. Show the computations that lead to your answer.

   point, slope

   $y-y_1=m(x-x_1)$

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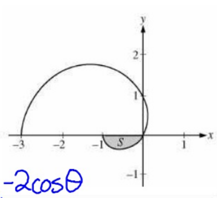
Question 4

The graph of the polar curve $r = 1 - 2\cos\theta$ for $0 \le \theta \le \pi$ is shown above. Let S be the shaded region in the third quadrant bounded by the curve and the x-axis.

(a) Write an integral expression for the area of S.

(b) Write expressions for $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ in terms of θ.

(c) Write an equation in terms of x and y for the line tangent to the graph of the polar curve at the point where $\theta = \frac{\pi}{2}$. Show the computations that lead to your answer.



$0=1-2\cos\theta$

$\cos\theta=\frac{1}{2}$

$\theta=\frac{\pi}{3}$

(a) $r(0) = -1$; $r(\theta) = 0$ when $\theta = \frac{\pi}{3}$.

Area of $S = \frac{1}{2}\int_0^{\pi/3}(1-2\cos\theta)^2\,d\theta$

$2:\begin{cases} 1: \text{limits and constant} \\ 1: \text{integrand} \end{cases}$

(b) $x = r\cos\theta$ and $y = r\sin\theta$

$\frac{dr}{d\theta} = 2\sin\theta$

$\frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta = \boxed{4\sin\theta\cos\theta - \sin\theta}$

$\frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta = \boxed{2\sin^2\theta + (1-2\cos\theta)\cos\theta}$

$4:\begin{cases} 1: \text{uses } x = r\cos\theta \text{ and } y = r\sin\theta \\ 1: \frac{dr}{d\theta} \\ 2: \text{answer} \end{cases}$

$2\sin\theta\cos\theta$
$-(1-2\cos\theta)\sin\theta$

$x=(1-2\cos\theta)\cos\theta$
$y=(1-2\cos\theta)\sin\theta$

(c) When $\theta = \frac{\pi}{2}$, we have $x = 0$, $y = 1$.

$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \left.\frac{dy/d\theta}{dx/d\theta}\right|_{\theta=\frac{\pi}{2}} = -2$

The tangent line is given by $y = 1 - 2x$.

$y-1=-2(x-0)$

$3:\begin{cases} 1: \text{values for } x \text{ and } y \\ 1: \text{expression for } \frac{dy}{dx} \\ 1: \text{tangent line equation} \end{cases}$

$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ at $\theta=\frac{\pi}{2} \to -2$

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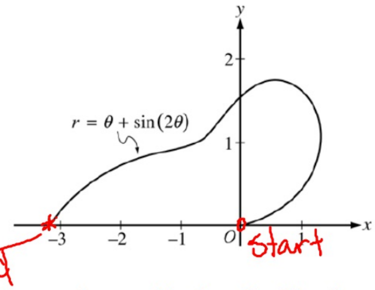

2. The curve above is drawn in the xy-plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \le \theta \le \pi$, where r is measured in meters and θ is measured in radians. The derivative of r with respect to θ is given by $\dfrac{dr}{d\theta} = 1 + 2\cos(2\theta)$.

$$\frac{1}{2}\int_0^{\pi} (\theta + \sin(2\theta))^2 \, d\theta = ?$$

(a) Find the area bounded by the curve and the x-axis.

(b) Find the angle θ that corresponds to the point on the curve with x-coordinate -2.

$$x = r\cos\theta = \underbrace{\boxed{(\theta + \sin 2\theta)\cos\theta}}_{Y_1} = \underbrace{\boxed{-2}}_{Y_2}$$

(c) For $\dfrac{\pi}{3} < \theta < \dfrac{2\pi}{3}$, $\dfrac{dr}{d\theta}$ is negative. What does this fact say about r? What does this fact say about the curve?

(d) Find the value of θ in the interval $0 \le \theta \le \dfrac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.