March 25

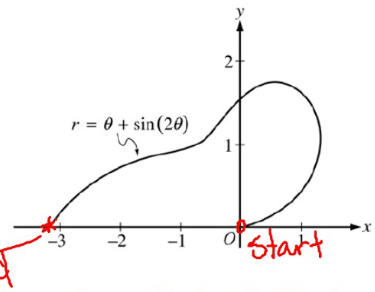
Mock Exam:
March 27th - 7:30 am
Room 5

How do you find a maximum or a minimum using the second derivative?

March 25

Students will verbally explain how to find the area enclosed by polar curves

(using the words:
radius, angle, arc...)



2. The curve above is drawn in the xy-plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \le \theta \le \pi$, where r is measured in meters and θ is measured in radians. The derivative of r with respect to θ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.

   (a) Find the area bounded by the curve and the x-axis. $\frac{1}{2}\int_0^{\pi} (\theta + \sin(2\theta))^2 \, d\theta = ?$

   (b) Find the angle θ that corresponds to the point on the curve with x-coordinate -2. $x = r\cos\theta = \underbrace{\boxed{(\theta + \sin 2\theta)\cos\theta}}_{Y_1} = \underbrace{\boxed{-2}}_{Y_2}$

   (c) For $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about r? What does this fact say about the curve?

   The radius is decreasing, the curve is getting closer to the origin

   (d) Find the value of θ in the interval $0 \le \theta \le \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.

maximum r-value

$1 + 2\cos 2\theta = 0$

$\cos 2\theta = \frac{-1}{2}$

$2\theta = \frac{2\pi}{3}$

$\theta = \frac{\pi}{3}$

CP		$\frac{\pi}{3}$
sign r'	+	−
behav. r	inc	dec

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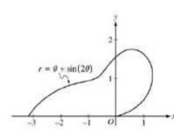
Question 2

The curve above is drawn in the xy-plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \le \theta \le \pi$, where r is measured in meters and θ is measured in radians. The derivative of r with respect to θ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.



(a) Find the area bounded by the curve and the x-axis.

(b) Find the angle θ that corresponds to the point on the curve with x-coordinate -2.

(c) For $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about r? What does this fact say about the curve?

(d) Find the value of θ in the interval $0 \le \theta \le \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.

(a) Area $= \frac{1}{2}\int_0^{\pi} r^2 \, d\theta$
$= \frac{1}{2}\int_0^{\pi} (\theta + \sin(2\theta))^2 \, d\theta = 4.382$

3 : { 1 : limits and constant; 1 : integrand; 1 : answer }

(b) $-2 = r\cos(\theta) = (\theta + \sin(2\theta))\cos(\theta)$
$\theta = 2.786$

2 : { 1 : equation; 1 : answer }

(c) Since $\frac{dr}{d\theta} < 0$ for $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, r is decreasing on this interval. This means the curve is getting closer to the origin.

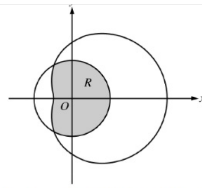
2 : { 1 : information about r; 1 : information about the curve }

(d) The only value in $\left[0, \frac{\pi}{2}\right]$ where $\frac{dr}{d\theta} = 0$ is $\theta = \frac{\pi}{3}$.

θ	r
0	0
$\frac{\pi}{3}$	1.913
$\frac{\pi}{2}$	1.571

The greatest distance occurs when $\theta = \frac{\pi}{3}$.

2 : { 1 : $\theta = \frac{\pi}{3}$ or 1.047; 1 : answer with justification }

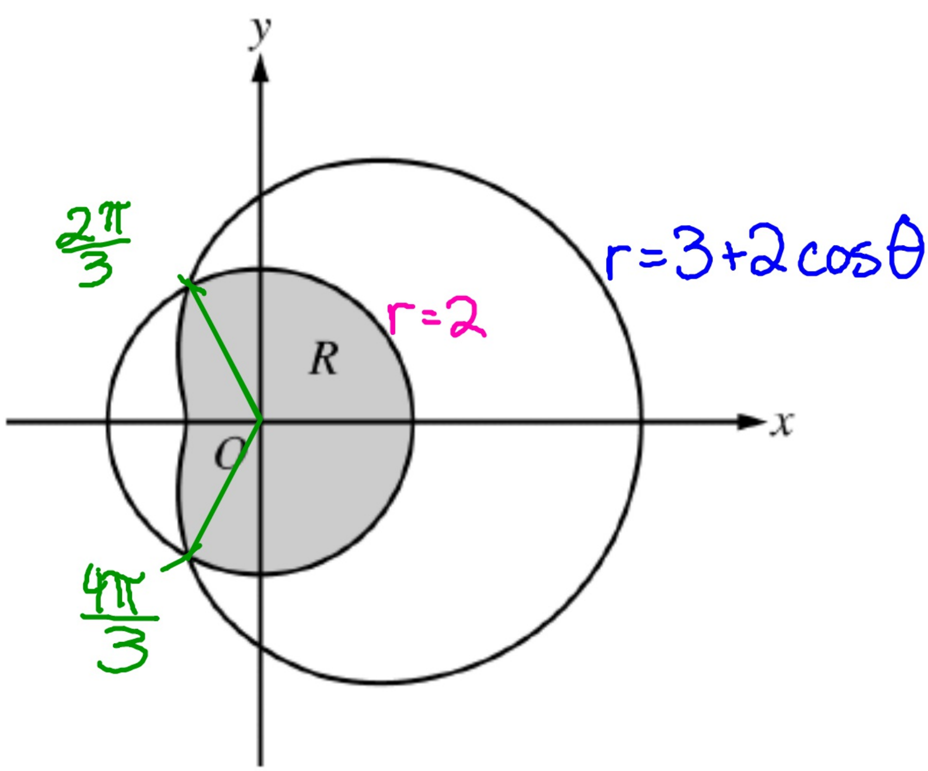


3. The graphs of the polar curves $r = 2$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.

(a) Let R be the region that is inside the graph of $r = 2$ and also inside the graph of $r = 3 + 2\cos\theta$, as shaded in the figure above. Find the area of R.

(b) A particle moving with nonzero velocity along the polar curve given by $r = 3 + 2\cos\theta$ has position $(x(t), y(t))$ at time t, with $\theta = 0$ when $t = 0$. This particle moves along the curve so that $\frac{dr}{dt} = \frac{dr}{d\theta}$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

(c) For the particle described in part (b), $\frac{dy}{dt} = \frac{dy}{d\theta}$. Find the value of $\frac{dy}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.



3. A particle is moving along a curve so that its position at time t is $(x(t), y(t))$, where $x(t) = t^2 - 4t + 8$ and $y(t)$ is not explicitly given. Both x and y are measured in meters, and t is measured in seconds. It is known that $\frac{dy}{dt} = te^{t-3} - 1$.

(a) Find the speed of the particle at time $t = 3$ seconds.

(b) Find the total distance traveled by the particle for $0 \le t \le 4$ seconds.

(c) Find the time t, $0 \le t \le 4$, when the line tangent to the path of the particle is horizontal. Is the direction of motion of the particle toward the left or toward the right at that time? Give a reason for your answer.

(d) There is a point with x-coordinate 5 through which the particle passes twice. Find each of the following.

  (i) The two values of t when that occurs

  (ii) The slopes of the lines tangent to the particle's path at that point

  (iii) The y-coordinate of that point, given $y(2) = 3 + \frac{1}{e}$