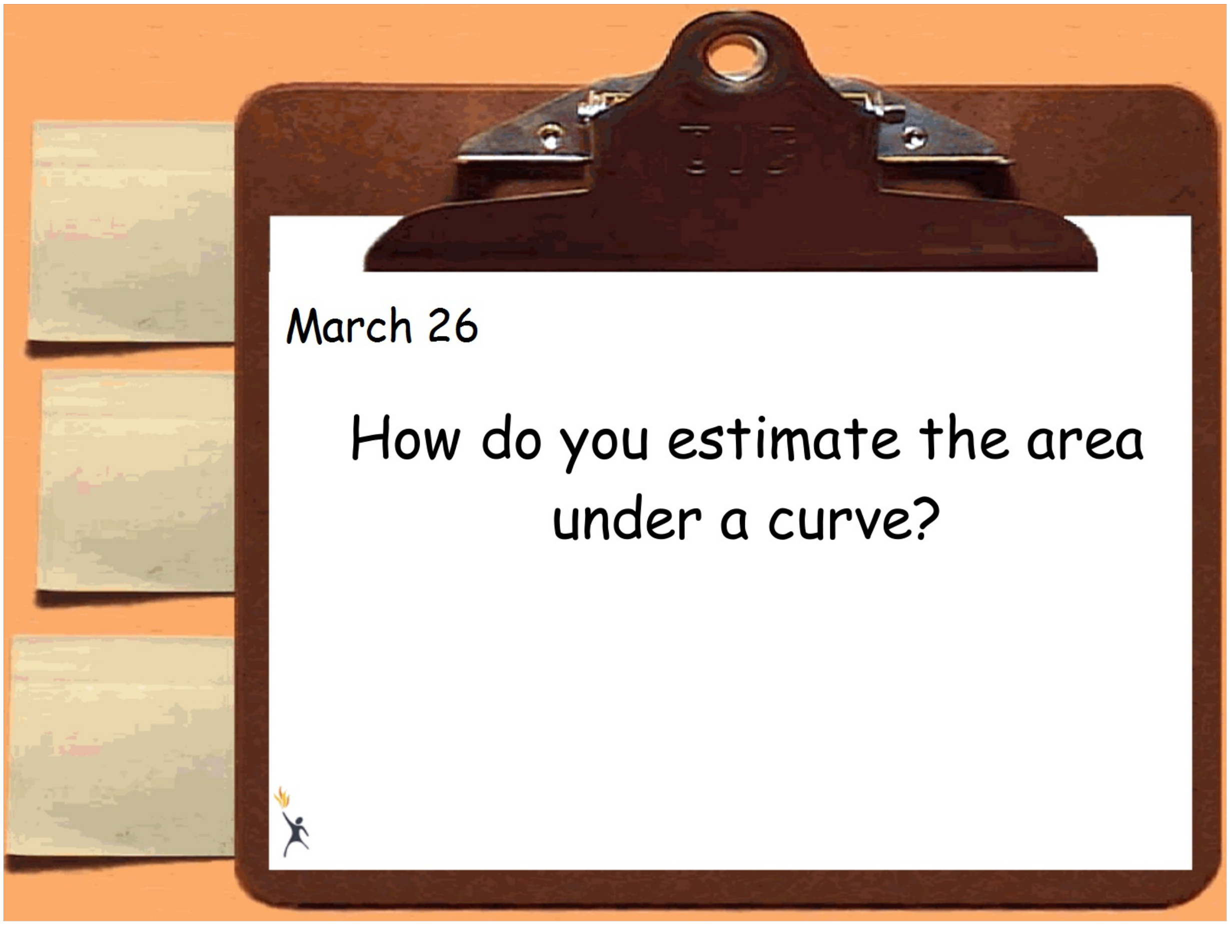
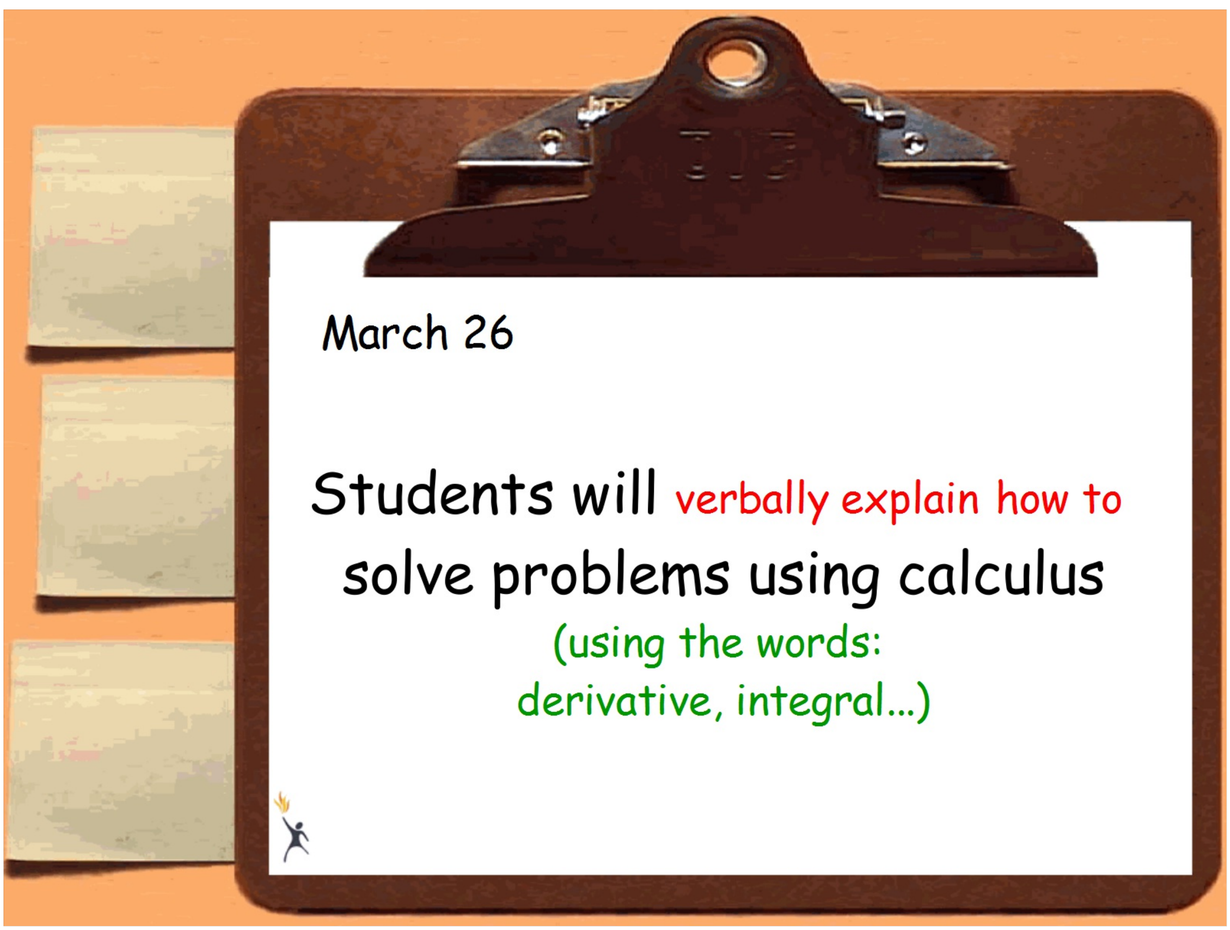

March 26
How do you estimate the area under a curve?


March 26
Students will verbally explain how to solve problems using calculus
(using the words:
derivative, integral...)

Mock Exam:

Thursday March 27th - 7:30 am

Room 5

(be there before 7:30)

If you need a calculator, come to room 115 before the test to get one.

(and return it when you finish the test)

Test Format

Part 1A: Multiple Choice - No Calculator

55 min - 28 questions #1-28

Part 1B: Multiple Choice - Calculator Allowed

50 min - 17 questions #75-92

Part 2A: Free Response - Calculator Allowed

30 min - 2 questions

Part 2B: Free Response - No Calculator

60 min - 4 questions

(can still work on part A questions, without a calculator)

Grades:

Test Grade based on questions over:

* parametric functions
* polar functions

Extra Credit for overall test score

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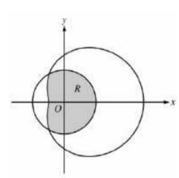
Question 3



The graphs of the polar curves $r = 2$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.

(a) Let R be the region that is inside the graph of $r = 2$ and also inside the graph of $r = 3 + 2\cos\theta$, as shaded in the figure above. Find the area of R.

(b) A particle moving with nonzero velocity along the polar curve given by $r = 3 + 2\cos\theta$ has position $(x(t), y(t))$ at time t, with $\theta = 0$ when $t = 0$. This particle moves along the curve so that $\frac{dr}{dt} = \frac{dr}{d\theta}$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

(c) For the particle described in part (b), $\frac{dy}{dt} = \frac{dy}{d\theta}$. Find the value of $\frac{dy}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

(a) $$\text{Area} = \frac{2}{3}\pi(2)^2 + \frac{1}{2}\int_{2\pi/3}^{4\pi/3}(3 + 2\cos\theta)^2\,d\theta = 10.370$$

$$4 : \begin{cases} 1 : \text{area of circular sector} \\ 2 : \text{integral for section of limaçon} \\ \quad 1 : \text{integrand} \\ \quad 1 : \text{limits and constant} \\ 1 : \text{answer} \end{cases}$$

(b) $$\left.\frac{dr}{dt}\right|_{\theta=\pi/3} = \left.\frac{dr}{d\theta}\right|_{\theta=\pi/3} = -1.732$$

The particle is moving closer to the origin, since $\frac{dr}{dt} < 0$ and $r > 0$ when $\theta = \frac{\pi}{3}$.

$$2 : \begin{cases} 1 : \left.\frac{dr}{dt}\right|_{\theta=\pi/3} \\ 1 : \text{interpretation} \end{cases}$$

(c) $$y = r\sin\theta = (3 + 2\cos\theta)\sin\theta$$
$$\left.\frac{dy}{dt}\right|_{\theta=\pi/3} = \left.\frac{dy}{d\theta}\right|_{\theta=\pi/3} = 0.5$$

The particle is moving away from the x-axis, since $\frac{dy}{dt} > 0$ and $y > 0$ when $\theta = \frac{\pi}{3}$.

$$3 : \begin{cases} 1 : \text{expression for } y \text{ in terms of } \theta \\ 1 : \left.\frac{dy}{dt}\right|_{\theta=\pi/3} \\ 1 : \text{interpretation} \end{cases}$$

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Question 3

A particle is moving along a curve so that its position at time t is $(x(t), y(t))$, where $x(t) = t^2 - 4t + 8$ and $y(t)$ is not explicitly given. Both x and y are measured in meters, and t is measured in seconds. It is known that $\frac{dy}{dt} = te^{t-3} - 1$.

(a) Find the speed of the particle at time $t = 3$ seconds.

(b) Find the total distance traveled by the particle for $0 \le t \le 4$ seconds.

(c) Find the time t, $0 \le t \le 4$, when the line tangent to the path of the particle is horizontal. Is the direction of motion of the particle toward the left or toward the right at that time? Give a reason for your answer.

(d) There is a point with x-coordinate 5 through which the particle passes twice. Find each of the following.

(i) The two values of t when that occurs

(ii) The slopes of the lines tangent to the particle's path at that point

(iii) The y-coordinate of that point, given $y(2) = 3 + \frac{1}{e}$

(a) Speed $= \sqrt{(x'(3))^2 + (y'(3))^2} = 2.828$ meters per second

1 : answer

(b) $x'(t) = 2t - 4$

Distance $= \int_0^4 \sqrt{(2t-4)^2 + \left(te^{t-3} - 1\right)^2}\, dt = 11.587$ or 11.588 meters

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 0$ when $te^{t-3} - 1 = 0$ and $2t - 4 \ne 0$

This occurs at $t = 2.20794$.

Since $x'(2.20794) > 0$, the particle is moving toward the right at time $t = 2.207$ or 2.208.

$3 : \begin{cases} 1 : \text{considers } \frac{dy}{dx} = 0 \\ 1 : t = 2.207 \text{ or } 2.208 \\ 1 : \text{direction of motion with reason} \end{cases}$

$t^2 - 4t + 8 = 5$

(d) $x(t) = 5$ at $t = 1$ and $t = 3$

At time $t = 1$, the slope is $\left.\frac{dy}{dx}\right|_{t=1} = \left.\frac{dy/dt}{dx/dt}\right|_{t=1} = 0.432.$

At time $t = 3$, the slope is $\left.\frac{dy}{dx}\right|_{t=3} = \left.\frac{dy/dt}{dx/dt}\right|_{t=3} = 1.$

$\boxed{y(1) = y(3)} = 3 + \frac{1}{e} + \int_2^3 \frac{dy}{dt}\, dt = 4$

$\int_2^3 y'\, dt = y(3) - y(2)$

$3 : \begin{cases} 1 : t = 1 \text{ and } t = 3 \\ 1 : \text{slopes} \\ 1 : y\text{-coordinate} \end{cases}$