March 6

How are position, velocity and acceleration related?

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Students will verbally explain how to use vectors to describe position, velocity and acceleration

(using the words: derivatives, integrals, magnitude...)

Vector Notation

Component Form:

$\langle x(t), y(t) \rangle$

Equation:

$x(t)\vec{i} + y(t)\vec{j}$

Position	Velocity	Acceleration
$\langle x(t), y(t) \rangle$	$\langle x'(t), y'(t) \rangle$	$\langle x''(t), y''(t) \rangle$

Speed

magnitude of velocity

$|v(t)|$

$\sqrt{(x'(t))^2 + (y'(t))^2}$

Direction

$\dfrac{v(t)}{|v(t)|}$

The position is given by $\langle \sin(2t), 2\cos t \rangle = \vec{r}(t)$

find the velocity & acceleration

$\vec{v}(t) = \langle 2\cos(2t), -2\sin t \rangle$

$\vec{a}(t) = \langle -4\sin(2t), -2\cos t \rangle$

$\vec{v}(t) = \langle e^t - t, e^t + t \rangle$

find the position vector if at $t=0$ the position is $\langle 1, 2 \rangle$

x-position

$$\int e^t - t\,dt = e^t - \frac{t^2}{2} + C$$

$1 = e^0 - \frac{0^2}{2} + C$

$1 = 1 - 0 + C \rightarrow C = 0$

$$\left\langle e^t - \frac{t^2}{2}, e^t + \frac{t^2}{2} + 1 \right\rangle$$

y-position

$$\int e^t + t\,dt = e^t + \frac{t^2}{2} + C$$

$2 = e^0 + \frac{0^2}{2} + C$

$2 = 1 + 0 + C \rightarrow C = 1$

Arc Length

$$= \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

Distance Traveled / Parametric Arc Length

$$= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

The position vector is given by $\langle \sin(3t), \ln(t)+t^2 \rangle$

find the speed at $t=2$

$v(t) = \langle 3\cos(3t), \frac{1}{t}+2t \rangle$

$\text{speed} = \sqrt{(3\cos(3\cdot 2))^2 + (\frac{1}{2}+2\cdot 2)^2}$

$= \sqrt{9\cos^2(6) + 4.5^2}$

$= 5.343$

find the distance traveled from $t=0.1$ to $t=2$

$D = \int_{0.1}^{2} \sqrt{(3\cos(3t))^2 + (\frac{1}{t}+2t)^2}\, dt$

$= 9.07196$