

Name \_\_\_\_\_

**PART I: Two-Layer Derivative:  $f(g(x))$**

Match each function with its derivative:

$$f(x) = \sin^6(x)$$

$$f'(x) = 3(9-x)^2 \cdot -1$$

$$f(x) = (9-x)^3$$

$$f'(x) = \cos(x^6) \cdot 6x^5$$

$$f(x) = \sin(x^6)$$

$$f'(x) = -\csc^2(3x) \cdot 3$$

$$f(x) = \cot(3x)$$

$$f'(x) = 6\sin^5(x) \cdot \cos(x)$$

What is the general (symbolic) rule for differentiating two-layer functions?

$$\frac{d}{dx} f(g(x)) =$$

**Find the derivatives of the functions below.**

1.  $f(x) = \cos(9x)$

2.  $f(x) = \sec(8x^2)$

3.  $f(x) = (17x^4 - 12)^8$

4.  $f(x) = \csc^3 x$

**Challenge! Given  $f'(x)$ , determine  $f(x)$ :**

$$f'(x) = \frac{1}{2}(x^3 + 5x)^{-1/2} \cdot (3x^2 + 5)$$

$$f(x) =$$

**PART II: Three-Layer Derivative:  $f(g(h(x)))$** 

Match each function with its derivative:

$f(x) = \sqrt{\sec(4x)}$

$f'(x) = 5 \tan^4(-2x^3 - 8) \cdot \sec^2(-2x^3 - 8) \cdot -6x^2$

$f(x) = \csc((4x)^8)$

$f'(x) = -\csc(4x)^8 \cot(4x)^8 \cdot 8(4x)^7 \cdot 4$

$f(x) = \tan^5(-2x^3 - 8)$

$f'(x) = 4 \cos^3(2 - 9x) \cdot -\sin(2 - 9x) \cdot -9$

$f(x) = \sin(\cos(7x))$

$f'(x) = \cos(\cos(7x)) \cdot -\sin(7x) \cdot 7$

$f(x) = \cos^4(2 - 9x)$

$f'(x) = \frac{1}{2} (\sec(4x))^{-\frac{1}{2}} \cdot \sec(4x) \tan(4x) \cdot 4$

What is the general (symbolic) rule for differentiating three-layer functions?

$$\frac{d}{dx} f(g(h(x))) =$$

**Find the derivatives of the functions below.**

1.  $f(x) = \cos^4 11x$

2.  $f(x) = \sqrt[3]{\sec x^2}$

3.  $f(x) = \cot(9x - x^2)^{10}$

4.  $f(x) = \cos(\csc(x^3))$

**Challenge! Given  $f'(x)$ , determine  $f(x)$ :**

$f'(x) = -\csc^2(4x^3 + 8x)^5 \cdot 5(4x^3 + 8x)^4 \cdot (12x^2 + 8)$

$f(x) =$

Write the chain rule out in words: