8 – Trig Formulas

Use trig formulas to rewrite each expression

(a) (Sum and Difference)
$\cos(6x - y)$

(b) (Double Angle)
$8\cos^2(x) - 4$

(c) (Power Reducing)
$7\cos^2(x)\tan^2(x)$

(d) (Half-Angle)
$$\frac{\sin(4h)}{1+\cos(4h)}$$

(e) (Product-to-Sum)
$\cos(8x)\sin(3x)$

(f) (Sum-to-Product)
$\cos(8y)+\cos(10x)$

8 – Trig Formulas – Answers

Use trig formulas to rewrite each expression

(a) (Sum and Difference)
$\cos(6x-y)$

$$\cos(6x-y)=\cos(6x)\cos(y)+\sin(6x)\sin(y)$$

(b) (Double Angle)
$8\cos^2(x)-4$

$$8\cos^2(x)-4=4\left(2\cos^2(x)-1\right)=4\cos(2x)$$

(c) (Power Reducing)
$7\cos^2(x)\tan^2(x)$

$$7\cos^2(x)\tan^2(x)=7\left(\frac{1+\cos(2x)}{2}\right)\left(\frac{1-\cos(2x)}{1+\cos(2x)}\right)$$

(d) (Half-Angle)
$\dfrac{\sin(4h)}{1+\cos(4h)}$

$$\frac{\sin(4h)}{1+\cos(4h)}=\tan\left(\frac{4h}{2}\right)=\tan(2h)$$

(e) (Product-to-Sum)
$\cos(8x)\sin(3x)$

$$\cos(8x)\sin(3x)=\frac{1}{2}\left[\sin(8x+3x)-\sin(8x-3x)\right]=\frac{1}{2}\left[\sin(11x)-\sin(5x)\right]$$

(f) (Sum-to-Product)
$\cos(8y)+\cos(10x)$

$$\cos(8y)+\cos(10x)=2\cos\left(\frac{8y+10x}{2}\right)\cos\left(\frac{8y-10x}{2}\right)=2\cos(4y+5x)\cos(4y-5x)$$

7 – Polar Graphs

(a) Give 3 sets of coordinate pairs
(at least one with a negative angle and at least one with a negative *r*-value)
that will graph the same point as $(4, 80^{o})$ when graphed on a Polar Grid.

(b) Graph each polar curve:

i. $r = 4\sin(2\theta)$

ii. $r = 2 - 3\cos(\theta)$

7 – Polar Graphs - Answers

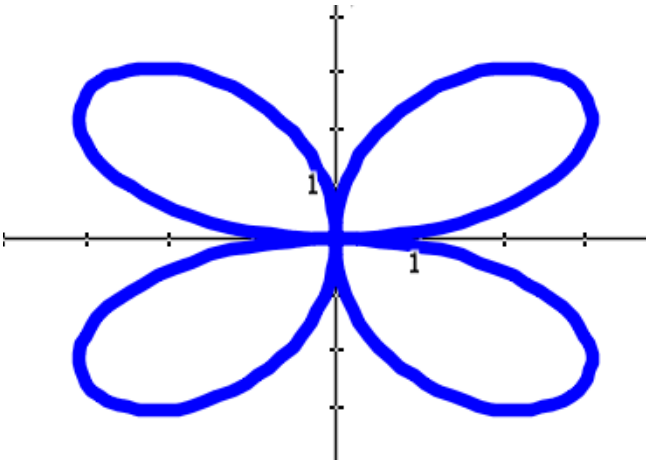
(a) Give 3 sets of coordinate pairs
(at least one with a negative angle and at least one with a negative *r*-value)
that will graph the same point as $(4, 80^o)$ when graphed on a Polar Grid.

$(4, 440^o)$
$(4, -280^o)$
$(-4, -100^o)$
$(-4, 260^o)$

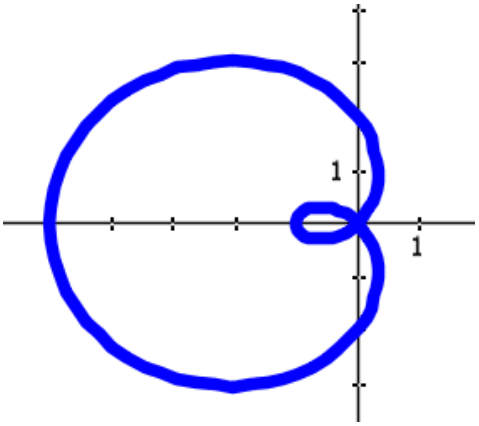
(and many more)

(b) Graph each polar curve:

$r = 4\sin(2\theta)$



$r = 2 - 3\cos(\theta)$



6 - Factoring

a. $x^2 + 6x + 8$

b. $x^2 - 2x - 24$

c. $2x^2 - 11x + 12$

6 – Factoring - Answers

a. $x^2 + 6x + 8$

$2 + 4 = 6$
$2 \times 4 = 8$

$x^2 + 2x + 4x + 8$
$x(x+2) + 4(x+2)$
$(x+2)(x+4)$

b. $x^2 - 2x - 24$

$-6 + 4 = -2$
$-6 \times 4 = -24$

$x^2 - 6x + 4x - 24$
$x(x-6) + 4(x-6)$
$(x-6)(x+4)$

c. $2x^2 - 11x + 12$

$-8 + -3 = -11$
$-8 \times -3 = 24$

$2x^2 - 8x - 3x + 12$
$2x(x-4) - 3(x-4)$
$(x-4)(2x-3)$

5 – Solving Trig Equations

Solve each equation for x on the interval $[0, 2\pi]$

a. $\sqrt{3}\csc x + 2 = 0$

b. $\cos^2 x \cot x = \cos^2 x$

c. $\cos^2 x + \sin x = 1$

5 - Solving Trig Equations - Answers

Solve each equation for x on the interval $[0, 2\pi]$

a. $\sqrt{3}\csc x + 2 = 0$

$$\sqrt{3}\csc x + 2 = 0$$
$$-2 = -2$$
$$\sqrt{3}\csc x = -2$$
$$\frac{\sqrt{3}\csc x}{\sqrt{3}} = \frac{-2}{\sqrt{3}}$$
$$\csc x = \frac{-2}{\sqrt{3}}$$
$$\csc^{-1}(\csc x) = \csc^{-1}\left(\frac{-2}{\sqrt{3}}\right)$$
$$x = \frac{4\pi}{3}, \frac{5\pi}{3}$$

b. $\cos^2 x \cot x = \cos^2 x$

$$\cos^2 x \cot x = \cos^2 x$$
$$-\cos^2 x = -\cos^2 x$$
$$\cos^2 x \cot x - \cos^2 x = 0$$
$$\cos^2 x(\cot x - 1) = 0$$

$\cos^2 x = 0$

$$\sqrt{\cos^2 x} = \sqrt{0}$$
$$\cos x = 0$$
$$\cos^{-1}(\cos x) = \cos^{-1}(0)$$
$$x = \cos^{-1}(0)$$
$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$\cot x - 1 = 0$

$$+1 \quad +1$$
$$\cot x = 1$$
$$\cot^{-1}(\cot x) = \cot^{-1}(1)$$
$$x = \cot^{-1}(1)$$
$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

c. $\cos^2 x + \sin x = 1$

$$\cos^2 x + \sin x = 1$$
$$\cos^2 x + \sin x = \cos^2 x + \sin^2 x$$
$$-\cos^2 x - \sin^2 x = -\cos^2 x - \sin^2 x$$
$$\sin x - \sin^2 x = 0$$
$$\sin x(1 - \sin x) = 0$$

$\sin x = 0$

$$\sin^{-1}(\sin x) = \sin^{-1}(0)$$
$$x = \sin^{-1}(0)$$
$$x = 0, \pi$$
$$=$$
$$=$$

$1 - \sin x = 0$

$$+\sin x \quad +\sin x$$
$$1 = \sin x$$
$$\sin^{-1}(1) = \sin^{-1}(\sin x)$$
$$\sin^{-1}(1) = x$$
$$\frac{\pi}{2} = x$$

4 – Exponential and Log Functions

Solve each equation below for x

(a) $7e^{3x} = 24$

(b) $4\ln(x-2)-12 = 20$

(c) $5^{2x-10} = 72$

4 – Exponential and Log Functions – Answers

Solve each equation below for x

(a) $7e^{3x} = 24$

$$\frac{7e^{3x}}{7} = \frac{24}{7}$$
$$e^{3x} = \frac{24}{7}$$
$$\ln\left(e^{3x}\right) = \ln\left(\frac{24}{7}\right)$$
$$3x = \ln\left(\frac{24}{7}\right)$$
$$\frac{3x}{3} = \frac{\ln\left(\frac{24}{7}\right)}{3}$$
$$x = \frac{\ln\left(\frac{24}{7}\right)}{3} = 0.4107$$

(b) $4\ln(x-2) - 12 = 20$

$$4\ln(x-2) - 12 = 20$$
$$+12 = +12$$
$$4\ln(x-2) = 32$$
$$\frac{4\ln(x-2)}{4} = \frac{32}{4}$$
$$\ln(x-2) = 8$$
$$e^{\ln(x-2)} = e^{8}$$
$$x - 2 = e^{8}$$
$$+2 = +2$$
$$x = e^{8} + 2 = 2982.96$$

(c) $5^{2x-10} = 72$

$$5^{2x-10} = 72$$
$$\ln\left(5^{2x-10}\right) = \ln(72)$$
$$(2x-10)\ln(5) = \ln(72)$$
$$\frac{(2x-10)\ln(5)}{\ln(5)} = \frac{\ln(72)}{\ln(5)}$$
$$2x - 10 == \frac{\ln(72)}{\ln(5)}$$
$$+10 = +10$$
$$2x == \frac{\ln(72)}{\ln(5)} + 10$$
$$\frac{2x}{2} == \frac{\frac{\ln(72)}{\ln(5)} + 10}{2}$$
$$x = \frac{\frac{\ln(72)}{\ln(5)} + 10}{2} = 6.328$$

3 – Inverse Trig Functions

Give the exact value for each trig function below:

(a) $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$

(b) $\arccos\left(-\frac{\sqrt{3}}{2}\right)$

(c) $\arctan\left(-\frac{1}{\sqrt{3}}\right)$

(d) $\sin^{-1}\left(-\frac{1}{2}\right)$

(e) $\tan^{-1}(1)$

(f) $\cos^{-1}(0)$

3 – Inverse Trig Functions – Answers

Give the exact value for each trig function below:

(a) $\arcsin\left(-\frac{\sqrt{2}}{2}\right)=-\frac{\pi}{4}$

(b) $\arccos\left(-\frac{\sqrt{3}}{2}\right)=\frac{5\pi}{6}$

(c) $\arctan\left(-\frac{1}{\sqrt{3}}\right)=-\frac{\pi}{6}$

(d) $\sin^{-1}\left(-\frac{1}{2}\right)=$

$$\sin^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6},\cdots$$
$$=\frac{7\pi}{6}+2n\pi \text{ and } \frac{11\pi}{6}+2n\pi$$

(e) $\tan^{-1}(1)$

$$\tan^{-1}(1)=\frac{\pi}{4},\frac{5\pi}{4},\frac{9\pi}{4},\cdots$$
$$=\frac{\pi}{4}+n\pi$$

(f) $\cos^{-1}(0)$

$$\cos^{-1}(0)=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},\cdots$$
$$=\frac{\pi}{2}+n\pi$$

2 – Graphing Trig functions

The height of the water in the harbor rose to a maximum height of 15 feet at 6:00 pm and then dropped to a minimum level of 3 feet at 3:00 am. Assume that the water level can be modeled by a sine function.

(a) Write an equation that represents the height *h* of the water *t* hours after **noon** on the first day.
(b) Sketch and label a graph of your function in part (a)

2 – Graphing Trig functions - Answers

The height of the water in the harbor rose to a maximum height of 15 feet at 6:00 pm and then dropped to a minimum level of 3 feet at 3:00 am. Assume that the water level can be modeled by a sine function.

(a) Write an equation that represents the height *h* of the water *t* hours after **noon** on the first day.

Vertical Shift = $\frac{15+3}{2} = 9$

Amplitude = 15 – 9 = 6 (or 9 – 3 = 6)

Period:

6:00 pm to 3:00 am = 9 hours

9 hours x 2 = 18 hours

Period = 18 hours

Horizontal Shift:

$\frac{18}{4} = 4.5$

High Point is at 6:00 pm → Cycle starts 4.5 hours earlier at 1:30 pm

Horizontal Shift = 1.5 hours after noon

B-value:

$$\text{period} = \frac{2\pi}{B}$$

$$18 = \frac{2\pi}{B} \quad \rightarrow\rightarrow \quad 18B = 2\pi$$

$$B = \frac{2\pi}{18} = \frac{\pi}{9}$$

C-value:

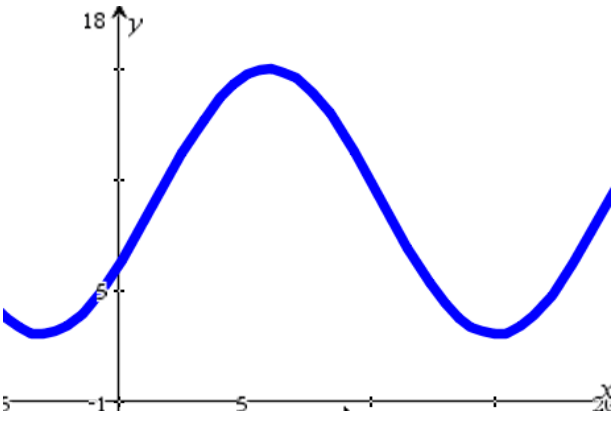
$$\text{horizontal shift} = \frac{-C}{B}$$

$$1.5 = \frac{-C}{\frac{\pi}{9}} \quad \rightarrow\rightarrow \quad 1.5\left(\frac{\pi}{9}\right) = -C$$

$$C = \frac{-1.5\pi}{9} = \frac{-\pi}{6}$$

Equation: $h = 6\sin\left(\frac{\pi}{9}x - \frac{\pi}{6}\right) + 9$

(b) Sketch and label a graph of your function in part (a)



1 – Unit Circle Values

Give the exact value for each trig function below:

(A) $\sin\left(\frac{4\pi}{3}\right)$

(B) $\cos\left(\frac{7\pi}{6}\right)$

(C) $\sec\left(\frac{7\pi}{4}\right)$

(D) $\tan\left(\frac{11\pi}{3}\right)$

(E) $\csc\left(-\frac{13\pi}{6}\right)$

(F) $\cot\left(\frac{19\pi}{4}\right)$

1 – Unit Circle Values – Answers

Give the exact value for each trig function below:

(A) $\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

(B) $\cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

(C) $\sec\left(\frac{7\pi}{4}\right) = \frac{2}{\sqrt{2}} = \sqrt{2}$

(D) $\tan\left(\frac{11\pi}{3}\right)$

$$\frac{11\pi}{3} - 2\pi = \frac{11\pi}{3} - \frac{6\pi}{3} = \frac{5\pi}{3}$$

$$\tan\left(\frac{11\pi}{3}\right) = \tan\left(\frac{5\pi}{3}\right) = -\sqrt{3}$$

(E) $\csc\left(-\frac{13\pi}{6}\right)$

$$-\frac{13\pi}{6} + 2\pi = -\frac{13\pi}{6} + \frac{12\pi}{6} = -\frac{\pi}{6}$$
$$-\frac{\pi}{6} + 2\pi = -\frac{\pi}{6} + \frac{12\pi}{6} = \frac{11\pi}{6}$$
$$\csc\left(-\frac{13\pi}{6}\right) = \csc\left(\frac{11\pi}{6}\right) = -2$$

(F) $\cot\left(\frac{19\pi}{4}\right)$

$$\frac{19\pi}{4} - 2\pi = \frac{19\pi}{4} - \frac{8\pi}{4} = \frac{11\pi}{4}$$
$$\frac{11\pi}{4} - 2\pi = \frac{11\pi}{4} - \frac{8\pi}{4} = \frac{3\pi}{4}$$
$$\cot\left(\frac{19\pi}{4}\right) = \cot\left(\frac{3\pi}{4}\right) = -1$$