In Exercises 5–10, plot the point given in polar coordinates and find the corresponding rectangular coordinates of the point.

5. $(4, -\pi/3)$
6. $(-1, -3\pi/4)$
7. $(0, -7\pi/6)$
8. $(32, 5\pi/2)$
9. $(\sqrt{2}, 2.36)$
10. $(-3, -1.57)$

In Exercises 11–14, use a graphing utility to find the rectangular coordinates of the point given in polar coordinates.

11. $(2, 3\pi/4)$
12. $(-2, 7\pi/6)$
13. $(-4.5, 1.3)$
14. $(8.25, 3.5)$

In Exercises 15–24, the rectangular coordinates of a point are given. Plot the point and find *two* sets of polar coordinates of the point for $0 \le \theta \le 2\pi$.

15. $(1, 1)$
16. $(0, -5)$
17. $(-6, 0)$
18. $(-3, -3)$
19. $(-3, 4)$
20. $(3, -1)$
21. $(-\sqrt{3}, -\sqrt{3})$
22. $(2, 0)$
23. $(4, 6)$
24. $(5, 12)$

In Exercises 25–30, use a graphing utility to find one set of polar coordinates of the point given in rectangular coordinates.

25. $(3, -2)$
26. $(-4, 1)$
27. $(\sqrt{3}, 2)$
28. $(3\sqrt{2}, 3\sqrt{2})$
29. $\left(\frac{5}{2}, \frac{4}{3}\right)$
30. $(0, -5)$

True or False? **In Exercises 31 and 32, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

31. If (r_1, θ_1) and (r_2, θ_2) represent the same point on the polar coordinate system, then $|r_1| = |r_2|$.
32. If (r, θ_1) and (r, θ_2) represent the same point on the polar coordinate system, then $\theta_1 = \theta_2 + 2\pi n$, for some integer n.

In Exercises 33–46, convert the rectangular equation to polar form.

33. $x^2 + y^2 = 9$
34. $x^2 + y^2 = a^2$
35. $x^2 + y^2 - 2ax = 0$
36. $x^2 + y^2 - 2ay = 0$
37. $y = 4$
38. $y = b$
39. $x = 10$
40. $x = a$
41. $3x - y + 2 = 0$
42. $4x + 7y - 2 = 0$
43. $xy = 4$
44. $y = x$
45. $(x^2 + y^2)^2 - 9(x^2 - y^2) = 0$
46. $y^2 - 8x - 16 = 0$

In Exercises 47–56, convert the polar equation to rectangular form.

47. $r = 4 \sin \theta$
48. $r = 4 \cos \theta$
49. $\theta = \dfrac{\pi}{6}$
50. $r = 4$
51. $r = 2 \csc \theta$
52. $r^2 = \sin 2\theta$
53. $r = 2 \sin 3\theta$
54. $r = \dfrac{1}{1 - \cos \theta}$
55. $r = \dfrac{6}{2 - 3 \sin \theta}$
56. $r = \dfrac{6}{2 \cos \theta - 3 \sin \theta}$

6.8 Exercises

In Exercises 1–6, identify the type of polar graph.

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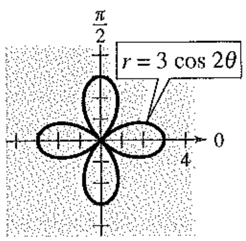

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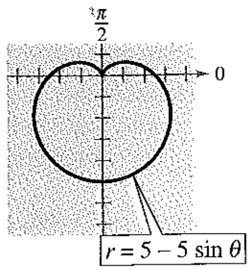

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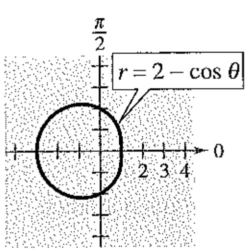

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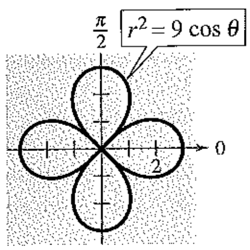

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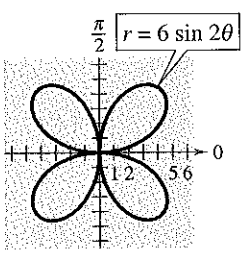

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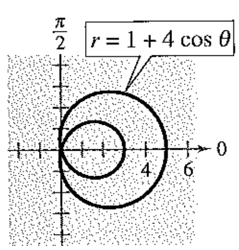
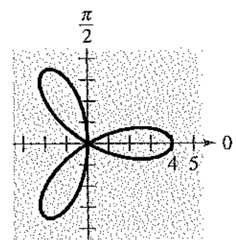


In Exercises 7–12, test for symmetry with respect to $\theta = \pi/2$, the polar axis, and the pole.

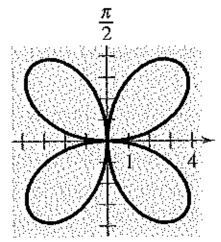
7. $r = 10 + 6\cos\theta$
8. $r = 16\cos 3\theta$
9. $r = \dfrac{2}{1 + \sin\theta}$
10. $r = 6\sin\theta$
11. $r = 4\sec\theta\csc\theta$
12. $r^2 = 25\sin 2\theta$

In Exercises 13–16, find the maximum value of $|r|$ and any zeros of r.

13. $r = 10(1 - \sin\theta)$
14. $r = 6 + 12\cos\theta$
15. $r = 4\cos 3\theta$



16. $r = 5\sin 2\theta$



In Exercises 17–46, sketch the graph of the polar equation.

17. $r = 5$
18. $r = 2$
19. $r = \dfrac{\pi}{6}$
20. $r = -\dfrac{\pi}{4}$
21. $r = 3\sin\theta$
22. $r = 3\cos\theta$
23. $r = 3(1 - \cos\theta)$
24. $r = 2(1 - \sin\theta)$
25. $r = 4(1 + \sin\theta)$
26. $r = 1 + \cos\theta$
27. $r = 3 - 2\cos\theta$
28. $r = 5 - 4\sin\theta$
29. $r = 2 + \sin\theta$
30. $r = 4 + 3\cos\theta$
31. $r = 2 + 4\sin\theta$
32. $r = 1 - 2\cos\theta$
33. $r = 3 - 4\cos\theta$
34. $r = 2(1 - 2\sin\theta)$
35. $r = 2\cos 3\theta$
36. $r = -\sin 5\theta$
37. $r = 3\sin 2\theta$
38. $r = 3\cos 2\theta$
39. $r = 2\sec\theta$
40. $r = 3\csc\theta$
41. $r = \dfrac{3}{\sin\theta - 2\cos\theta}$
42. $r = \dfrac{6}{2\sin\theta - 3\cos\theta}$
43. $r^2 = 4\cos 2\theta$
44. $r^2 = 4\sin\theta$
45. $r = \dfrac{\theta}{2}$
46. $r = \theta$

In Exercises 47–54, use a graphing utility to graph the polar equation.

47. $r = 6\cos\theta$
48. $r = \dfrac{\theta}{4}$
49. $r = 3(2 - \sin\theta)$
50. $r = \cos 2\theta$

51. $r = 4 \sin\theta \cos^2\theta$

52. $r = 2\cos(3\theta - 2)$

53. $r = 2\csc\theta + 5$

54. $r = 2 - \sec\theta$

In Exercises 55–62, use a graphing utility to graph the polar equation. Find an interval for θ for which the graph is traced *only once*.

55. $r = 3 - 4\cos\theta$

56. $r = 2(1 - 2\sin\theta)$

57. $r = 2 + \sin\theta$

58. $r = 4 + 3\cos\theta$

59. $r = 2\cos\left(\frac{3\theta}{2}\right)$

60. $r = 3\sin\left(\frac{5\theta}{2}\right)$

61. $r^2 = 4\sin 2\theta$

62. $r^2 = \frac{1}{\theta}$

In Exercises 63–66, use a graphing utility to graph the polar equation and show that the indicated line is an asymptote of the graph.

	Name of Graph	*Polar Equation*	*Asymptote*
63.	Conchoid	$r = 2 - \sec\theta$	$x = -1$
64.	Conchoid	$r = 2 + \csc\theta$	$y = 1$
65.	Hyperbolic spiral	$r = \frac{2}{\theta}$	$y = 2$
66.	Strophoid	$r = 2\cos 2\theta \sec\theta$	$x = -2$

67. ***Exploration*** Sketch the graph of $r = 4\sin\theta$ over each interval. Describe the part of the graph obtained in each case.

(a) $0 \le \theta \le \frac{\pi}{2}$ (b) $\frac{\pi}{2} \le \theta \le \pi$

(c) $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ (d) $\frac{\pi}{4} \le \theta \le \frac{3\pi}{4}$

68. ***Graphical Reasoning*** Use a graphing utility to graph the polar equation

$$r = 6[1 + \cos(\theta - \phi)]$$

for (a) $\phi = 0$, (b) $\phi = \pi/4$, and (c) $\phi = \pi/2$. Use the graphs to describe the effect of the angle ϕ. Write the equation as a function of $\sin\theta$ for part (c).

69. The graph of $r = f(\theta)$ is rotated about the pole through an angle ϕ. Show that the equation of the rotated graph is $r = f(\theta - \phi)$.

70. Consider the graph of $r = f(\sin\theta)$.

(a) Show that if the graph is rotated counterclockwise $\pi/2$ radians about the pole, the equation of the rotated graph is $r = f(-\cos\theta)$.

(b) Show that if the graph is rotated counterclockwise π radians about the pole, the equation of the rotated graph is $r = f(-\sin\theta)$.

(c) Show that if the graph is rotated counterclockwise $3\pi/2$ radians about the pole, the equation of the rotated graph is $r = f(\cos\theta)$.

In Exercises 71–74, use the results of Exercises 69 and 70.

71. Write an equation for the limaçon $r = 2 - \sin\theta$ after it has been rotated by the given amount.

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) $\frac{3\pi}{2}$

72. Write an equation for the rose curve $r = 2\sin 2\theta$ after it has been rotated by the given amount.

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) π

73. Sketch the graph of each equation.

(a) $r = 1 - \sin\theta$ (b) $r = 1 - \sin\left(\theta - \frac{\pi}{4}\right)$

74. Sketch the graph of each equation.

(a) $r = 3\sec\theta$ (b) $r = 3\sec\left(\theta - \frac{\pi}{4}\right)$

(c) $r = 3\sec\left(\theta + \frac{\pi}{3}\right)$ (d) $r = 3\sec\left(\theta - \frac{\pi}{2}\right)$

75. ***Exploration*** Use a graphing utility to graph and identify $r = 2 + k\cos\theta$ for $k = 0, 1, 2,$ and 3.

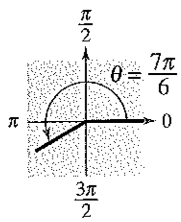
76. ***Exploration*** Consider the equation $r = 3\sin k\theta$.

(a) Use a graphing utility to graph the equation for $k = 1.5$. Find the interval for θ for which the graph is traced only once.

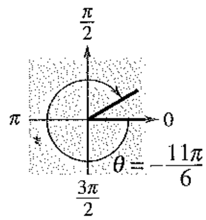
(b) Use a graphing utility to graph the equation for $k = 2.5$. Find the interval for θ for which the graph is traced only once.

(c) Is it possible to find an interval for θ for which the graph is traced only once for any rational number k? Explain.

16. (a)



(b)



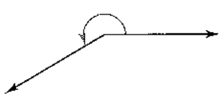
17. (a) $-9\pi/4$ (b) $-2\pi/15$
18. (a) $8\pi/9$ (b) $8\pi/45$

In Exercises 19–20, find (if possible) the complement and supplement of the angle.

19. (a) $\pi/3$ (b) $3\pi/4$
20. (a) 1 (b) 2

In Exercises 21–24, estimate the angle in degrees.

21.



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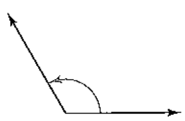

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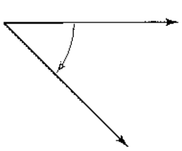

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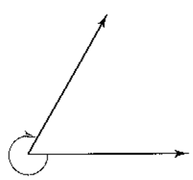


In Exercises 25–28, determine the quadrant in which the angle lies.

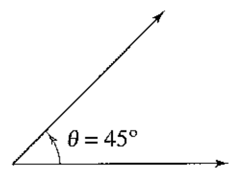
25. (a) 130° (b) 285°
26. (a) 8.3° (b) 257° 30′
27. (a) −132° 50′ (b) −336°
28. (a) −260° (b) −3.4°

In Exercises 29–32, sketch the angle in standard position.

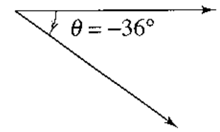
29. (a) 30° (b) 150°
30. (a) −270° (b) −120°
31. (a) 405° (b) −480°
32. (a) 750° (b) −600°

In Exercises 33–36, determine two coterminal angles (one positive and one negative) for the given angle. Give your answers in degrees.

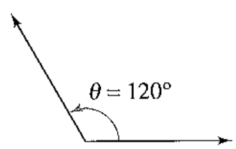
33. (a)



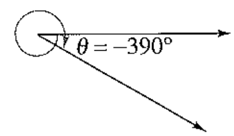
(b)



34. (a)



(b)



35. (a) 300° (b) 740°
36. (a) −420° (b) 230°

In Exercises 37 and 38, find (if possible) the complement and supplement of the angle.

37. (a) 18° (b) 115°
38. (a) 79° (b) 150°

In Exercises 39–42, express the angle in radian measure as a multiple of π. (Do not use a calculator.)

39. (a) 30° (b) 150°
40. (a) 315° (b) 120°
41. (a) −20° (b) −240°
42. (a) −270° (b) 144°

In Exercises 43–46, express the angle in degree measure. (Do not use a calculator.)

43. (a) $3\pi/2$ (b) $7\pi/6$
44. (a) $-7\pi/12$ (b) $\pi/9$
45. (a) $7\pi/3$ (b) $-11\pi/30$
46. (a) $11\pi/6$ (b) $34\pi/15$

Name: ____________________ period: _____

1) Solve: $(2+x)^2 = 64$

In Exercises 47–54, convert the measure from degrees to radians. Round to three decimal places.

47. $115°$ **48.** $87.4°$
49. $-216.35°$ **50.** $-48.27°$
51. $532°$ **52.** $0.54°$
53. $-0.83°$ **54.** $345°$

In Exercises 55–62, convert the measure from radians to degrees. Round to three decimal places.

55. $\pi/7$ **56.** $5\pi/11$
57. $15\pi/8$ **58.** 6.5π
59. -4.2π **60.** 4.8
61. -2 **62.** -0.57

In Exercises 63–66, convert to decimal degree form.

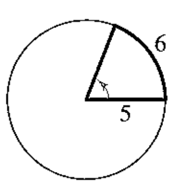
63. (a) $54° 45'$ (b) $-128° 30'$
64. (a) $245° 10'$ (b) $2° 12'$
65. (a) $85° 18' 30''$ (b) $330° 25''$
66. (a) $-135° 36''$ (b) $-408° 16' 20''$

In Exercises 67–70, convert to D° M′ S″ form.

67. (a) $240.6°$ (b) $-145.8°$
68. (a) $-345.12°$ (b) 0.45
69. (a) 2.5 (b) -3.58
70. (a) -0.355 (b) 0.7865

In Exercises 71–74, find the angle in radians.

71.



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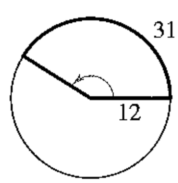

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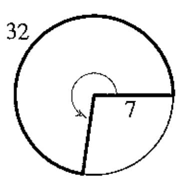

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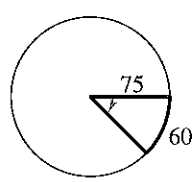


In Exercises 75–78, find the radian measure of the central angle of a circle of the given radius that intercepts an arc of the given length.

	Radius	*Arc Length*
75.	15 inches	4 inches
76.	16 feet	10 feet
77.	14.5 centimeters	25 centimeters
78.	80 kilometers	160 kilometers

In Exercises 79–82, find the length of the arc on a circle of the given radius intercepted by the given central angle.

	Radius	*Central Angle*
79.	15 inches	$180°$
80.	9 feet	$60°$
81.	6 meters	2 radians
82.	40 centimeters	$3\pi/4$ radians

Distance Between Cities **In Exercises 83–86, find the distance between the cities. Assume that earth is a sphere of radius 4000 miles and the cities are on the same meridian (one city is due north of the other).**

	City	*Latitude*
83.	Dallas, Texas	$32° 47' 9''$N
	Omaha, Nebraska	$41° 15' 42''$N
84.	San Francisco, California	$37° 46' 39''$N
	Seattle, Washington	$47° 36' 32''$N
85.	Miami, Florida	$25° 46' 37''$N
	Erie, Pennsylvania	$42° 7' 15''$N
86.	Johannesburg, South Africa	$26° 10'$S
	Jerusalem, Israel	$31° 47'$N

87. ***Difference in Latitudes*** Assuming that earth is a sphere of radius 6378 kilometers, what is the difference in latitude of two cities, one of which is 600 kilometers due north of the other?

88. ***Difference in Latitudes*** Assuming that earth is a sphere of radius 6378 kilometers, what is the difference in latitude of two cities, one of which is 800 kilometers due north of the other?

Name: ______________________ period: _____

1) Solve: $(2+x)^2 = 64$

WARM UP

Simplify the expression.

1. $\dfrac{1/2}{-\sqrt{3}/2}$

2. $\dfrac{\sqrt{2}/2}{-\sqrt{2}/2}$

Find a coterminal angle in the interval $[0, 2\pi]$.

3. $\dfrac{8\pi}{3}$

4. $-\dfrac{\pi}{4}$

5. Convert 30° to radians

6. Convert 135° to radians.

7. Convert $\dfrac{\pi}{3}$ to degrees

8. Convert $\dfrac{3\pi}{2}$ to degrees.

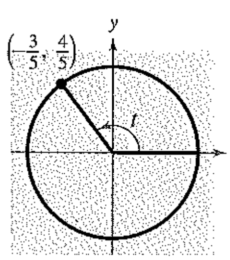
9. Determine the circumference of a circle with radius 1.

10. Determine arc length of a semicircle with radius 1.

1.2 Exercises

In Exercises 1–4, find the six trigonometric functions of t that correspond to the point on the unit circle.

1.



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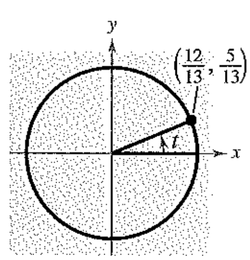

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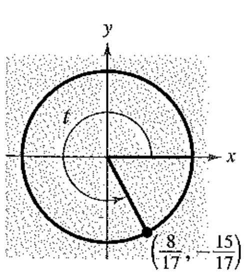

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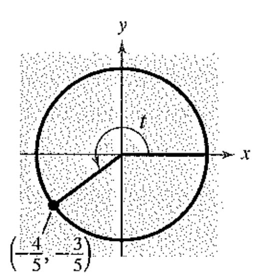


In Exercises 5–12, find the point (x, y) on the unit circle that corresponds to the real number t.

5. $t = \dfrac{\pi}{4}$

6. $t = \dfrac{\pi}{3}$

7. $t = \dfrac{5\pi}{6}$

8. $t = \dfrac{5\pi}{4}$

9. $t = \dfrac{4\pi}{3}$

10. $t = \dfrac{11\pi}{6}$

11. $t = \dfrac{3\pi}{2}$

12. $t = \pi$

In Exercises 13–24, evaluate (if possible) the sine, cosine, and tangent of the real number.

13. $t = \dfrac{\pi}{4}$

14. $t = -\dfrac{\pi}{4}$

15. $t = -\dfrac{\pi}{6}$

16. $t = \dfrac{\pi}{3}$

17. $t = -\dfrac{5\pi}{4}$

18. $t = -\dfrac{5\pi}{6}$

19. $t = \dfrac{11\pi}{6}$

20. $t = \dfrac{2\pi}{3}$

21. $t = \dfrac{4\pi}{3}$

22. $t = \dfrac{7\pi}{4}$

23. $t = -\dfrac{3\pi}{2}$

24. $t = -2\pi$

In Exercises 25–30, evaluate (if possible) the six trigonometric functions of the real number.

25. $t = \dfrac{3\pi}{4}$

26. $t = -\dfrac{2\pi}{3}$

27. $t = \dfrac{\pi}{2}$

28. $t = \dfrac{3\pi}{2}$

29. $t = -\dfrac{4\pi}{3}$

30. $t = -\dfrac{11\pi}{6}$

In Exercises 31–38, evaluate the trigonometric functions using its period as an aid.

31. $\sin 3\pi$

32. $\cos 3\pi$

33. $\cos \dfrac{8\pi}{3}$

34. $\sin \dfrac{9\pi}{4}$

35. $\cos \dfrac{19\pi}{6}$

36. $\sin\left(-\dfrac{13\pi}{6}\right)$

37. $\sin\left(-\dfrac{9\pi}{4}\right)$

38. $\cos\left(-\dfrac{8\pi}{3}\right)$

In Exercises 39–44, use the value of the trigonometric function to evaluate the indicated functions.

39. $\sin t = \frac{1}{3}$
(a) $\sin(-t)$
(b) $\csc(-t)$

40. $\sin(-t) = \frac{2}{5}$
(a) $\sin t$
(b) $\csc t$

41. $\cos(-t) = -\frac{7}{8}$
(a) $\cos t$
(b) $\sec(-t)$

42. $\cos t = -\frac{3}{4}$
(a) $\cos(-t)$
(b) $\sec(-t)$

43. $\sin t = \frac{4}{5}$
(a) $\sin(\pi - t)$
(b) $\sin(t + \pi)$

44. $\cos t = \frac{4}{5}$
(a) $\cos(\pi - t)$
(b) $\cos(t + \pi)$

In Exercises 45–54, use a calculator to evaluate the expression. Round to four decimal places.

45. $\sin \dfrac{\pi}{4}$

46. $\tan \pi$

47. $\cos(-3)$

48. $\cot 1$

49. $\cos(-1.7)$

50. $\csc 2.3$

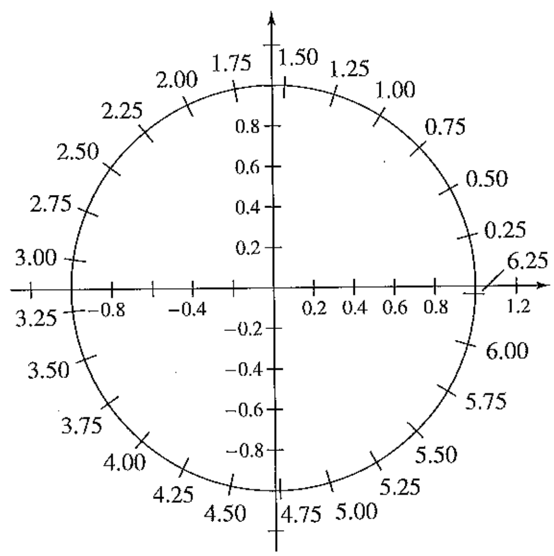
51. $\csc 0.8$

52. $\sec 1.8$

53. $\sec 22.8$

54. $\sin(-0.9)$

In Exercises 55–58, use the figure and a straightedge.



55. Approximate the trigonometric function.
(a) $\sin 5$
(b) $\cos 2$

56. Approximate the trigonometric function.
(a) $\sin 0.75$
(b) $\cos 2.5$

57. Approximate t where $0 \le t < 2\pi$.
(a) $\sin t = 0.25$
(b) $\cos t = -0.25$

Name: ______________________ period: _____

1) Solve: $(2+x)^2 = 64$

WARM UP

Find the distance between the points.

1. $(3, 8), (1, 4)$
2. $(5, 2), (2, -7)$
3. $(-4, 0), (2, 8)$
4. $(-3, -3), (0, 0)$

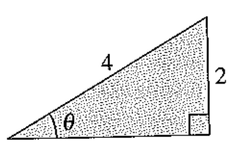
Perform the operations. (Round your answer to two decimal places.)

5. 0.300×4.125
6. 7.30×43.50
7. $\dfrac{151.5}{2.40}$
8. $\dfrac{3740}{28.0}$
9. $\dfrac{19{,}500}{0.007}$
10. $\dfrac{(10.5)(3401)}{1240}$

1.3 Exercises

In Exercises 1–4, find the exact values of the six trigonometric functions of the angle θ given in the figure. (Use the Pythagorean Theorem to find the third side of the triangle.)

1.



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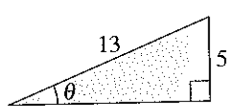

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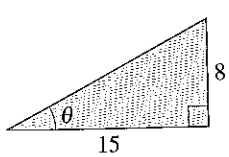

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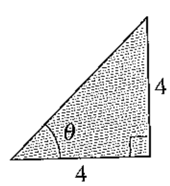
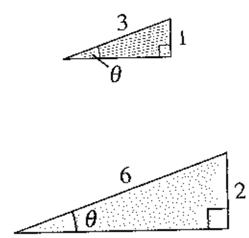


In Exercises 5–8, find the exact values of the six trigonometric functions of the angle θ for each of the triangles. Explain why the function values are the same.

5.



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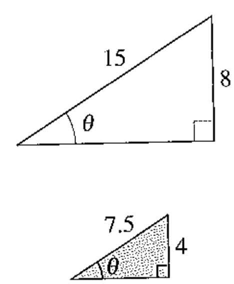

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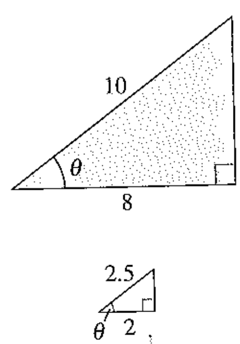

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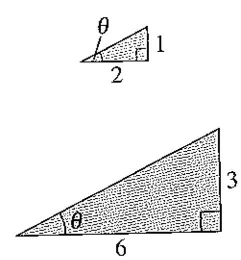


In Exercises 9–16, sketch a right triangle corresponding to the trigonometric function of the acute angle θ. Use the Pythagorean Theorem to determine the third side, and then find the other five trigonometric functions of θ.

9. $\sin\theta = \frac{2}{3}$ **10.** $\cot\theta = 5$

11. $\sec\theta = 2$ **12.** $\cos\theta = \frac{5}{7}$

13. $\tan\theta = 3$ **14.** $\csc\theta = \frac{17}{4}$

15. $\cot\theta = \frac{3}{2}$ **16.** $\sin\theta = \frac{3}{8}$

In Exercises 17–22, use the given function values and the appropriate trigonometric identities (including the relationship between a trigonometric function and its cofunction of a complementary angle) to find the indicated trigonometric functions.

17. $\sin 60° = \dfrac{\sqrt{3}}{2}$, $\cos 60° = \dfrac{1}{2}$

(a) $\tan 60°$ (b) $\sin 30°$

(c) $\cos 30°$ (d) $\cot 60°$

18. $\sin 30° = \dfrac{1}{2}$, $\tan 30° = \dfrac{\sqrt{3}}{3}$

(a) $\csc 30°$ (b) $\cot 60°$

(c) $\cos 30°$ (d) $\cot 30°$

19. $\csc\theta = 3$, $\sec\theta = \dfrac{3\sqrt{2}}{4}$

(a) $\sin\theta$ (b) $\cos\theta$

(c) $\tan\theta$ (d) $\sec(90° - \theta)$

20. $\sec\theta = 5$, $\tan\theta = 2\sqrt{6}$

(a) $\cos\theta$ (b) $\cot\theta$

(c) $\cot(90° - \theta)$ (d) $\sin\theta$

21. $\cos\alpha = \frac{1}{4}$

(a) $\sec\alpha$ (b) $\sin\alpha$

(c) $\cot\alpha$ (d) $\sin(90° - \alpha)$

22. $\tan\beta = 5$

(a) $\cot\beta$ (b) $\cos\beta$

(c) $\tan(90° - \beta)$ (d) $\csc\beta$

In Exercises 23–32, use trigonometric identities to transform one side of the equation into the other.

23. $\tan\theta\cot\theta = 1$ **24.** $\cos\theta\sec\theta = 1$

25. $\tan\alpha\cos\alpha = \sin\alpha$ **26.** $\cot\alpha\sin\alpha = \cos\alpha$

27. $(1 + \cos\theta)(1 - \cos\theta) = \sin^2\theta$

28. $(1 + \sin\theta)(1 - \sin\theta) = \cos^2\theta$

29. $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

30. $\sin^2\theta - \cos^2\theta = 2\sin^2\theta - 1$

31. $\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \csc\theta\sec\theta$

32. $\dfrac{\tan\beta + \cot\beta}{\tan\beta} = \csc^2\beta$

In Exercises 33–36, evaluate the trigonometric function by memory or by constructing an appropriate triangle for the given special angle.

33. (a) $\cos 60°$ (b) $\tan\dfrac{\pi}{6}$

34. (a) $\csc 30°$ (b) $\sin\dfrac{\pi}{4}$

35. (a) $\cot 45°$ (b) $\cos 45°$

36. (a) $\sin\dfrac{\pi}{3}$ (b) $\csc 45°$

In Exercises 37–46, use a calculator to evaluate each function. Round your answers to four decimal places. (Be sure the calculator is in the correct mode.)

37. (a) $\sin 10°$ (b) $\cos 80°$

38. (a) $\tan 23.5°$ (b) $\cot 66.5°$

39. (a) $\sin 16.35°$ (b) $\csc 16.35°$

40. (a) $\cos 16°\,18'$ (b) $\sin 73°\,56'$

41. (a) $\sec 42°\,12'$ (b) $\csc 48°\,7'$

42. (a) $\cos 4°\,50'\,15''$ (b) $\sec 4°\,50'\,15''$

43. (a) $\cot\dfrac{\pi}{16}$ (b) $\tan\dfrac{\pi}{16}$

44. (a) $\sec 0.75$ (b) $\cos 0.75$

45. (a) $\csc 1$ (b) $\tan\frac{1}{2}$

46. (a) $\sec\left(\dfrac{\pi}{2} - 1\right)$ (b) $\cot\left(\dfrac{\pi}{2} - \dfrac{1}{2}\right)$

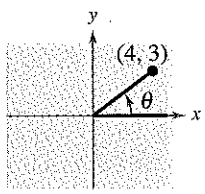
Name: ______________________ period: _____

1) Solve: $(2+x)^2 = 64$

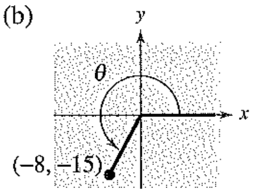
1.4 Exercises

In Exercises 1–4, determine the exact values of the six trigonometric functions of the angle θ.

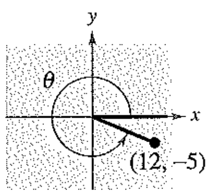
1. (a)



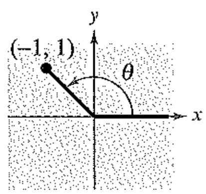
(b)



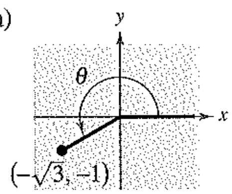
2. (a)



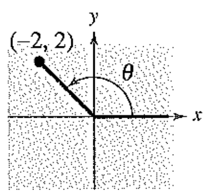
(b)



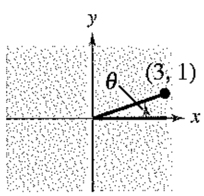
3. (a)



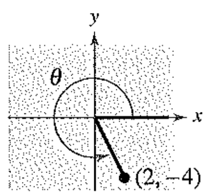
(b)



4. (a)



(b)



In Exercises 5–8, the point is on the terminal side of an angle in standard position. Determine the exact values of the six trigonometric functions of the angle.

5. (a) $(7, 24)$ (b) $(7, -24)$

6. (a) $(8, 15)$ (b) $(-9, -40)$

7. (a) $(-4, 10)$ (b) $(3, -5)$

8. (a) $(-5, -2)$ (b) $\left(-\frac{3}{2}, 3\right)$

In Exercises 9–12, state the quadrant in which θ lies.

9. (a) $\sin\theta < 0$ and $\cos\theta < 0$
(b) $\sin\theta > 0$ and $\cos\theta < 0$

10. (a) $\sin\theta > 0$ and $\cos\theta > 0$
(b) $\sin\theta < 0$ and $\cos\theta > 0$

11. (a) $\sin\theta > 0$ and $\tan\theta < 0$
(b) $\cos\theta > 0$ and $\tan\theta < 0$

12. (a) $\sec\theta > 0$ and $\cot\theta < 0$
(b) $\csc\theta < 0$ and $\tan\theta > 0$

In Exercises 13–22, find the values (if possible) of the six trigonometric functions of θ using the functional value and constraint.

	Functional Value	*Constraint*
13.	$\sin\theta = \frac{3}{5}$	θ lies in Quadrant II.
14.	$\cos\theta = -\frac{4}{5}$	θ lies in Quadrant III.
15.	$\tan\theta = -\frac{15}{8}$	$\sin\theta < 0$
16.	$\cos\theta = \frac{8}{17}$	$\tan\theta < 0$
17.	$\cot\theta = -3$	$\cos\theta > 0$
18.	$\csc\theta = 4$	$\cot\theta < 0$
19.	$\sec\theta = -2$	$\sin\theta > 0$
20.	$\cot\theta$ is undefined.	$\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$
21.	$\sin\theta = 0$	$\sec\theta = -1$
22.	$\tan\theta$ is undefined.	$\pi \le \theta \le 2\pi$

In Exercises 23–26, find the values (if possible) of the six trigonometric functions of θ if the terminal side of θ lies on the given line in the specified quadrant.

	Line	*Quadrant*
23.	$y = -x$	Quadrant II
24.	$y = \frac{1}{3}x$	Quadrant III
25.	$y = 2x$	Quadrant III
26.	$4x + 3y = 0$	Quadrant IV

Name: ______________________ period: ____

1) Solve: $(2+x)^2 = 64$

In Exercises 47–52, find the values of θ in degrees $(0° < \theta < 90°)$ and radians $(0 < \theta < \pi/2)$ without the aid of a calculator.

47. (a) $\sin\theta = \dfrac{1}{2}$ (b) $\csc\theta = 2$

48. (a) $\cos\theta = \dfrac{\sqrt{2}}{2}$ (b) $\tan\theta = 1$

49. (a) $\sec\theta = 2$ (b) $\cot\theta = 1$

50. (a) $\tan\theta = \sqrt{3}$ (b) $\cos\theta = \dfrac{1}{2}$

51. (a) $\csc\theta = \dfrac{2\sqrt{3}}{3}$ (b) $\sin\theta = \dfrac{\sqrt{2}}{2}$

52. (a) $\cot\theta = \dfrac{\sqrt{3}}{3}$ (b) $\sec\theta = \sqrt{2}$

In Exercises 53–56, find the values of θ in degrees $(0° < \theta < 90°)$ and radians $(0 < \theta < \pi/2)$ by using a calculator.

53. (a) $\sin\theta = 0.8191$ (b) $\cos\theta = 0.0175$

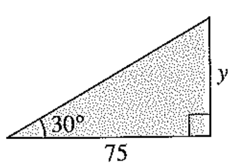
54. (a) $\cos\theta = 0.9848$ (b) $\cos\theta = 0.8746$

55. (a) $\tan\theta = 1.1920$ (b) $\tan\theta = 0.4663$

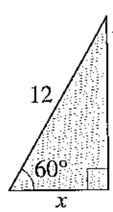
56. (a) $\sin\theta = 0.3746$ (b) $\cos\theta = 0.3746$

In Exercises 57–64, solve for x, y, or r, as indicated.

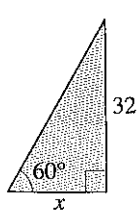
57. Solve for y.



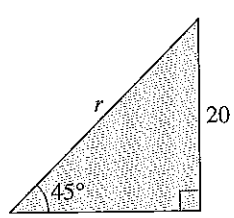
58. Solve for x.



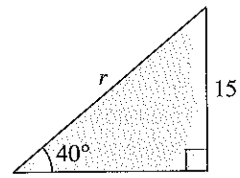
59. Solve for x.



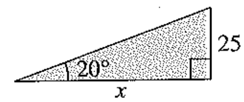
60. Solve for r.



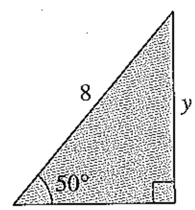
61. Solve for r.



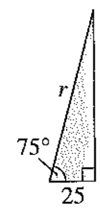
62. Solve for x.



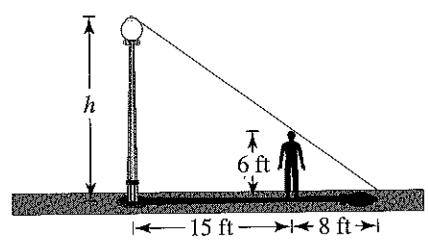
63. Solve for y.



64. Solve for r.



65. ***Height*** A 6-foot person standing 15 feet from a streetlight casts an 8-foot shadow (see figure). What is the height of the streetlight?



66. ***Height*** A 6-foot person walks from the base of a broadcasting tower directly toward the tip of the shadow cast by the tower. When the person is 132 feet from the tower and 3 feet from the tip of the tower's shadow, the person's shadow starts to appear beyond the tower's shadow.

(a) Draw a right triangle that gives a visual representation of the problem. Show the known quantities on the triangle and use a variable to indicate the height of the tower.

(b) Write an equation involving the unknown.

(c) What is the height of the tower?

WARM UP

Simplify the expression.

1. $\dfrac{2\pi}{1/3}$

2. $\dfrac{2\pi}{4\pi}$

Solve for x.

3. $2x - \dfrac{\pi}{3} = 0$

4. $2x - \dfrac{\pi}{3} = 2\pi$

5. $3\pi x + 6\pi = 0$

6. $3\pi x + 6\pi = 2\pi$

Evaluate the trigonometric function from memory.

7. $\sin \dfrac{\pi}{2}$

8. $\sin \pi$

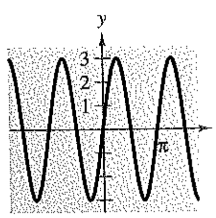
9. $\cos 0$

10. $\cos \dfrac{\pi}{2}$

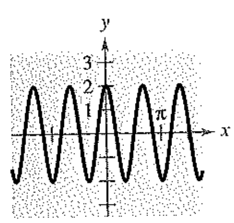
1.5 Exercises

In Exercises 1–14, find the period and amplitude.

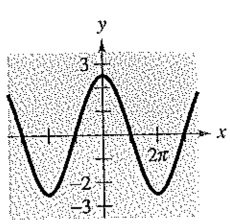
1. $y = 3 \sin 2x$



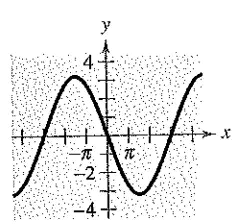
2. $y = 2 \cos 3x$



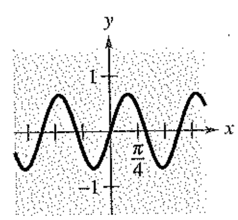
3. $y = \dfrac{5}{2} \cos \dfrac{x}{2}$



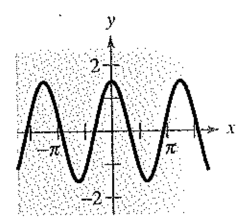
4. $y = -3 \sin \dfrac{x}{3}$



5. $y = \dfrac{2}{3} \sin \pi x$



6. $y = \dfrac{3}{2} \cos \dfrac{\pi x}{2}$



7. $y = -2 \sin x$

8. $y = -\cos \dfrac{2x}{3}$

9. $y = 3 \sin 10x$

10. $y = \frac{1}{3} \sin 8x$

11. $y = \dfrac{1}{2} \cos \dfrac{2x}{3}$

12. $y = \dfrac{5}{2} \cos \dfrac{x}{4}$

13. $y = 3 \sin 4\pi x$

14. $y = \dfrac{2}{3} \cos \dfrac{\pi x}{10}$

Name: ______________________ period: ____

In Exercises 15–22, describe the relationship between the graphs of f and g.

15. $f(x) = \sin x$
 $g(x) = \sin(x - \pi)$

16. $f(x) = \cos x$
 $g(x) = \cos(x + \pi)$

17. $f(x) = \cos 2x$
 $g(x) = -\cos 2x$

18. $f(x) = \sin 3x$
 $g(x) = \sin(-3x)$

19. $f(x) = \cos x$
 $g(x) = \cos 2x$

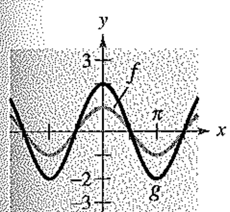
20. $f(x) = \sin x$
 $g(x) = \sin 3x$

21. $f(x) = \sin x$
 $g(x) = 2 + \sin x$

22. $f(x) = \cos 4x$
 $g(x) = -2 + \cos 4x$

In Exercises 23–26, describe the relationship between the graphs of f and g.

23.



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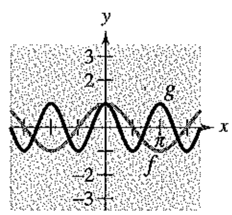

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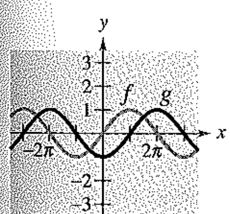

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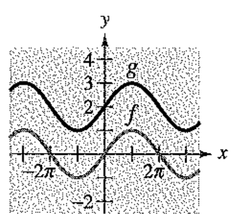


27. ***Essay*** Use a graphing utility to graph the function $y = a \sin x$ for $a = \frac{1}{2}$, $a = \frac{3}{2}$, and $a = -3$. Write a paragraph describing the changes in the graph corresponding to the specified changes in a.

28. ***Essay*** Use a graphing utility to graph the function $y = d + \sin x$ for $d = 2, d = 3.5$, and $d = -2$. Write a paragraph describing the changes in the graph corresponding to the specified changes in d.

29. ***Essay*** Use a graphing utility to graph the function $y = \sin bx$ for $b = \frac{1}{2}$, $b = \frac{3}{2}$, and $b = 4$. Write a paragraph describing the changes in the graph corresponding to the specified changes in b.

30. ***Essay*** Use a graphing utility to graph the function $y = \sin(x - c)$ for $c = 1$, $c = 3$, and $c = -2$. Write a paragraph describing the changes in the graph corresponding to the specified changes in c.

In Exercises 31–38, graph f and g on the same set of coordinate axes. (Include two full periods.)

31. $f(x) = -2 \sin x$
 $g(x) = 4 \sin x$

32. $f(x) = \sin x$
 $g(x) = \sin \dfrac{x}{3}$

33. $f(x) = \cos x$
 $g(x) = 1 + \cos x$

34. $f(x) = 2 \cos 2x$
 $g(x) = -\cos 4x$

35. $f(x) = -\dfrac{1}{2} \sin \dfrac{x}{2}$
 $g(x) = 3 - \dfrac{1}{2} \sin \dfrac{x}{2}$

36. $f(x) = 4 \sin \pi x$
 $g(x) = 4 \sin \pi x - 3$

37. $f(x) = 2 \cos x$
 $g(x) = 2 \cos(x + \pi)$

38. $f(x) = -\cos x$
 $g(x) = -\cos(x - \pi)$

Conjecture **In Exercises 39–42, graph f and g on the same set of coordinate axes. (Include two full periods.) Make a conjecture about the functions.**

39. $f(x) = \sin x, \quad g(x) = \cos\left(x - \dfrac{\pi}{2}\right)$

40. $f(x) = \sin x, \quad g(x) = -\cos\left(x + \dfrac{\pi}{2}\right)$

41. $f(x) = \cos x, \quad g(x) = -\sin\left(x - \dfrac{\pi}{2}\right)$

42. $f(x) = \cos x, \quad g(x) = -\cos(x - \pi)$

In Exercises 43–60, sketch the graph of the function. (Include two full periods.)

43. $y = -2 \sin 6x$

44. $y = -3 \cos 4x$

45. $y = \cos 2\pi x$

46. $y = \dfrac{3}{2} \sin \dfrac{\pi x}{4}$

47. $y = -\sin \dfrac{2\pi x}{3}$

48. $y = 10 \cos \dfrac{\pi x}{6}$

49. $y = \sin\left(x - \frac{\pi}{4}\right)$

50. $y = \frac{1}{2}\sin(x - \pi)$

51. $y = 3\cos(x + \pi)$

52. $y = 4\cos\left(x + \frac{\pi}{4}\right)$

53. $y = \frac{1}{10}\cos 60\pi x$

54. $y = -3 + 5\cos\frac{\pi t}{12}$

55. $y = 2 - \sin\frac{2\pi x}{3}$

56. $y = 2\cos x - 3$

57. $y = 3\cos(x + \pi) - 3$

58. $y = 4\cos\left(x + \frac{\pi}{4}\right) + 4$

59. $y = \frac{2}{3}\cos\left(\frac{x}{2} - \frac{\pi}{4}\right)$

60. $y = -3\cos(6x + \pi)$

In Exercises 61–68, use a graphing utility to graph the function. (Include two full periods.)

61. $y = -2\sin(4x + \pi)$

62. $y = -4\sin\left(\frac{2}{3}x - \frac{\pi}{3}\right)$

63. $y = \cos\left(2\pi x - \frac{\pi}{2}\right) + 1$

64. $y = 3\cos\left(\frac{\pi x}{2} + \frac{\pi}{2}\right) - 2$

65. $y = -0.1\sin\left(\frac{\pi x}{10} + \pi\right)$

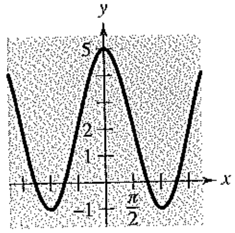
66. $y = 5\sin(\pi - 2x) + 10$

67. $y = 5\cos(\pi - 2x) + 2$

68. $y = \frac{1}{100}\sin 120\pi t$

Graphical Reasoning **In Exercises 69–72, find a and d for the function $f(x) = a\cos x + d$ so that the graph of f matches the figure.**

69.



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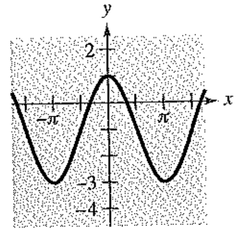

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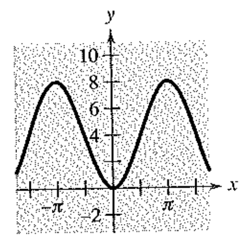

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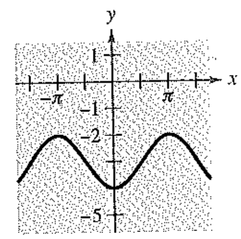
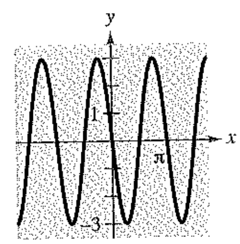


Graphical Reasoning **In Exercises 73–76, find a, b, and c for the function $y = a\sin(bx - c)$ so that the graph of f matches the figure.**

73.



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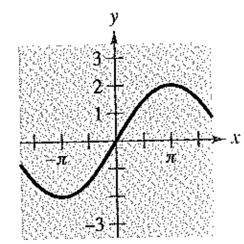

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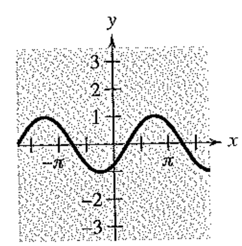

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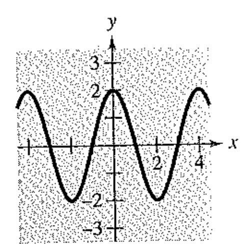


In Exercises 77–80, use a graphing utility to graph y_1 and y_2 in the interval $[-2\pi, 2\pi]$. Use the graphs to find real numbers x such that $y_1 = y_2$.

77. $y_1 = \sin x$

$y_2 = -\frac{1}{2}$

78. $y_1 = \cos x$

$y_2 = -1$

79. $y_1 = \cos x$

$y_2 = \frac{\sqrt{2}}{2}$

80. $y_1 = \sin x$

$y_2 = \frac{\sqrt{3}}{2}$

Name: ________________ period: ____

In Exercises 9–30, sketch the graph of the function. (Include two full periods.)

9. $y = \frac{1}{3}\tan x$
10. $y = \frac{1}{4}\tan x$
11. $y = \tan 2x$
12. $y = -3\tan \pi x$
13. $y = -\frac{1}{2}\sec x$
14. $y = \frac{1}{4}\sec x$
15. $y = \sec \pi x$
16. $y = 2\sec 4x$
17. $y = \sec \pi x - 1$
18. $y = -2\sec 4x + 2$
19. $y = \csc \frac{x}{2}$
20. $y = \csc \frac{x}{3}$
21. $y = \cot \frac{x}{2}$
22. $y = 3\cot \frac{\pi x}{2}$
23. $y = \frac{1}{2}\sec 2x$
24. $y = -\frac{1}{2}\tan x$
25. $y = \tan \frac{\pi x}{4}$
26. $y = \sec(x + \pi)$
27. $y = \csc(\pi - x)$
28. $y = \sec(\pi - x)$
29. $y = \frac{1}{4}\csc\left(x + \frac{\pi}{4}\right)$
30. $y = 2\cot\left(x + \frac{\pi}{2}\right)$

In Exercises 31–40, use a graphing utility to graph the function. (Include two full periods.)

31. $y = \tan \frac{x}{3}$
32. $y = -\tan 2x$
33. $y = -2\sec 4x$
34. $y = \sec \pi x$
35. $y = \tan\left(x - \frac{\pi}{4}\right)$
36. $y = -\csc(4x - \pi)$
37. $y = \frac{1}{4}\cot\left(x - \frac{\pi}{2}\right)$
38. $y = 0.1\tan\left(\frac{\pi x}{4} + \frac{\pi}{4}\right)$
39. $y = 2\sec(2x - \pi)$
40. $y = \frac{1}{3}\sec\left(\frac{\pi x}{2} + \frac{\pi}{2}\right)$

In Exercises 41–44, use a graph to solve the equation on the interval $[-2\pi, 2\pi]$.

41. $\tan x = 1$
42. $\cot x = -\sqrt{3}$
43. $\sec x = -2$
44. $\csc x = \sqrt{2}$

In Exercises 45 and 46, use the graph of the function to determine whether the function is even, odd, or neither.

45. $f(x) = \sec x$
46. $f(x) = \tan x$

47. ***Essay*** Describe the behavior of $f(x) = \tan x$ as x approaches $\pi/2$ from the left and from the right.

48. ***Essay*** Describe the behavior of $f(x) = \csc x$ as x approaches π from the left and from the right.

49. ***Graphical Reasoning*** Consider the functions

$$f(x) = 2\sin x \quad \text{and} \quad g(x) = \frac{1}{2}\csc x$$

on the interval $(0, \pi)$.

(a) Graph f and g in the same coordinate plane.
(b) Approximate the interval where $f > g$.
(c) Describe the behavior of each of the functions as x approaches π. How is the behavior of g related to the behavior of f as x approaches π?

50. ***Graphical Reasoning*** Consider the functions

$$f(x) = \tan \frac{\pi x}{2} \quad \text{and} \quad g(x) = \frac{1}{2}\sec \frac{\pi x}{2}$$

on the interval $(-1, 1)$.

(a) Use a graphing utility to graph f and g on the same viewing rectangle.
(b) Approximate the interval where $f < g$.
(c) Approximate the interval where $2f < 2g$. How does the result compare with that of part (b)? Explain.

In Exercises 51–54, use a graphing utility to graph the two equations on the same viewing rectangle. Determine analytically whether the expressions are equivalent.

51. $y_1 = \sin x \csc x, \quad y_2 = 1$
52. $y_1 = \sin x \sec x, \quad y_2 = \tan x$
53. $y_1 = \dfrac{\cos x}{\sin x}, \quad y_2 = \cot x$
54. $y_1 = \sec^2 x - 1, \quad y_2 = \tan^2 x$

49. $y = \sin\left(x - \frac{\pi}{4}\right)$

50. $y = \frac{1}{2}\sin(x - \pi)$

51. $y = 3\cos(x + \pi)$

52. $y = 4\cos\left(x + \frac{\pi}{4}\right)$

53. $y = \frac{1}{10}\cos 60\pi x$

54. $y = -3 + 5\cos\frac{\pi t}{12}$

55. $y = 2 - \sin\frac{2\pi x}{3}$

56. $y = 2\cos x - 3$

57. $y = 3\cos(x + \pi) - 3$

58. $y = 4\cos\left(x + \frac{\pi}{4}\right) + 4$

59. $y = \frac{2}{3}\cos\left(\frac{x}{2} - \frac{\pi}{4}\right)$

60. $y = -3\cos(6x + \pi)$

In Exercises 61–68, use a graphing utility to graph the function. (Include two full periods.)

61. $y = -2\sin(4x + \pi)$

62. $y = -4\sin\left(\frac{2}{3}x - \frac{\pi}{3}\right)$

63. $y = \cos\left(2\pi x - \frac{\pi}{2}\right) + 1$

64. $y = 3\cos\left(\frac{\pi x}{2} + \frac{\pi}{2}\right) - 2$

65. $y = -0.1\sin\left(\frac{\pi x}{10} + \pi\right)$

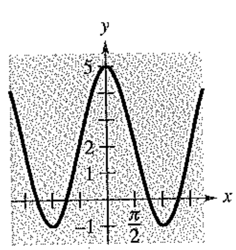
66. $y = 5\sin(\pi - 2x) + 10$

67. $y = 5\cos(\pi - 2x) + 2$

68. $y = \frac{1}{100}\sin 120\pi t$

Graphical Reasoning **In Exercises 69–72, find a and d for the function $f(x) = a\cos x + d$ so that the graph of f matches the figure.**

69.



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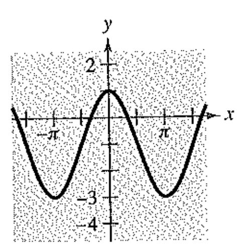

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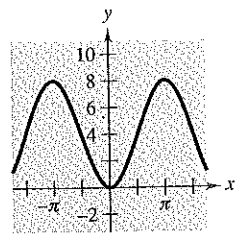

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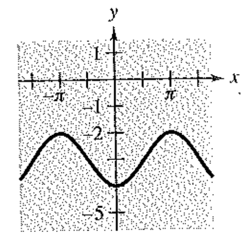
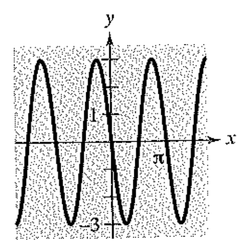


Graphical Reasoning **In Exercises 73–76, find a, b, and c for the function $y = a\sin(bx - c)$ so that the graph of f matches the figure.**

73.



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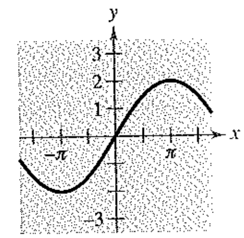

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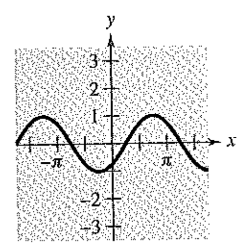

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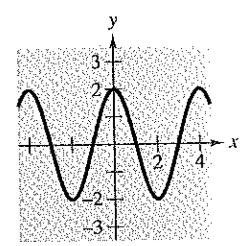

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77. $y_1 = \sin x$
$y_2 = -\frac{1}{2}$

78. $y_1 = \cos x$
$y_2 = -1$

79. $y_1 = \cos x$
$y_2 = \frac{\sqrt{2}}{2}$

80. $y_1 = \sin x$
$y_2 = \frac{\sqrt{3}}{2}$

WARM UP

Evaluate the trigonometric function from memory.

1. $\tan 0$
2. $\cos \frac{\pi}{4}$
3. $\tan \frac{\pi}{4}$
4. $\cot \frac{\pi}{2}$
5. $\sin \pi$
6. $\cos \frac{\pi}{2}$

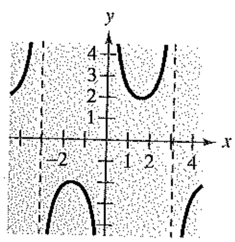
Sketch the graph of the function. (Include two full periods.)

7. $y = -2 \cos 2x$
8. $y = 3 \sin \frac{x}{4}$
9. $y = \frac{3}{2} \sin 2\pi x$
10. $y = -2 \cos \frac{\pi x}{2}$

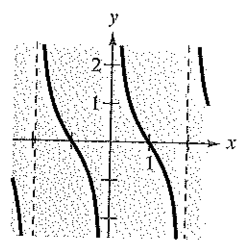
1.6 Exercises

In Exercises 1–8, match the function with its graph. State the period of the function. [The graphs are labeled (a), (b), (c), (d), (e), (f), (g), and (h).]

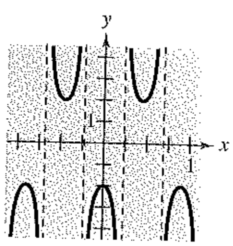
(a)



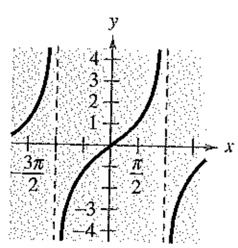
(b)



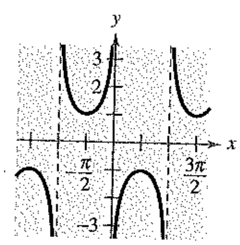
(c)



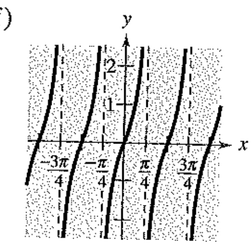
(d)



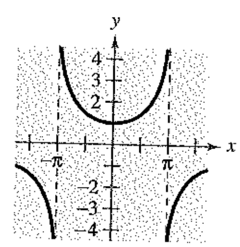
(e)



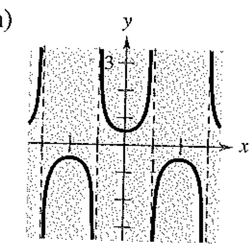
(f)



(g)



(h)



1. $y = \sec \frac{x}{2}$
2. $y = \tan \frac{x}{2}$
3. $y = \tan 2x$
4. $y = 2 \csc x$
5. $y = \cot \frac{\pi x}{2}$
6. $y = \frac{1}{2} \sec \frac{\pi x}{2}$
7. $y = -\csc x$
8. $y = -2 \sec 2\pi x$

1.7 Exercises

1. ***True or False?*** Explain your reasoning.

$$\sin\frac{5\pi}{6}=\frac{1}{2} \Longrightarrow \arcsin\frac{1}{2}=\frac{5\pi}{6}$$

2. ***True or False?*** Explain your reasoning.

$$\tan\frac{5\pi}{4}=1 \Longrightarrow \arctan 1=\frac{5\pi}{4}$$

In Exercises 3–18, evaluate the expression without the aid of a calculator.

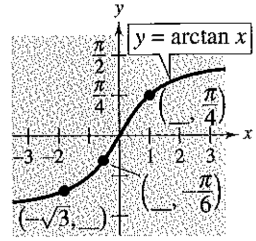
3. $\arcsin\frac{1}{2}$
4. $\arcsin 0$
5. $\arccos\frac{1}{2}$
6. $\arccos 0$
7. $\arctan\frac{\sqrt{3}}{3}$
8. $\arctan(-1)$
9. $\arccos\left(-\frac{\sqrt{3}}{2}\right)$
10. $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$
11. $\arctan(-\sqrt{3})$
12. $\arctan\sqrt{3}$
13. $\arccos\left(-\frac{1}{2}\right)$
14. $\arcsin\frac{\sqrt{2}}{2}$
15. $\arcsin\frac{\sqrt{3}}{2}$
16. $\arctan\left(-\frac{\sqrt{3}}{3}\right)$
17. $\arctan 0$
18. $\arccos 1$

In Exercises 19–30, use a calculator to approximate the expression. (Round your result to two decimal places.)

19. $\arccos 0.28$
20. $\arcsin 0.45$
21. $\arcsin(-0.75)$
22. $\arccos(-0.7)$
23. $\arctan(-3)$
24. $\arctan 15$
25. $\arcsin 0.31$
26. $\arccos 0.26$
27. $\arccos(-0.41)$
28. $\arcsin(-0.125)$
29. $\arctan 0.92$
30. $\arctan 2.8$

In Exercises 31 and 32, determine the missing coordinates of the points on the graph of the function.

31.



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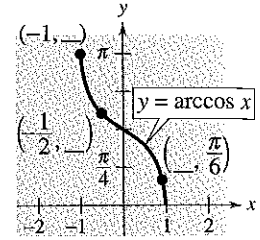
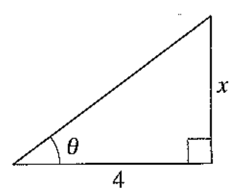


In Exercises 33 and 34, use a graphing utility to graph f, g, and $y = x$ on the same viewing rectangle to verify geometrically that g is the inverse of f. (Be sure to restrict the domain of f properly.)

33. $f(x)=\tan x,\quad g(x)=\arctan x$
34. $f(x)=\sin x,\quad g(x)=\arcsin x$

In Exercises 35–38, use an inverse trigonometric function to write θ as a function of x.

35.



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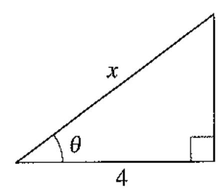

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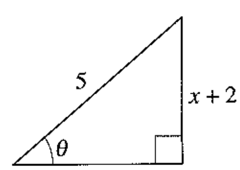

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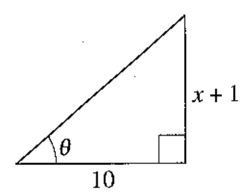


WARM UP

Evaluate the expression and round to two decimal places.

1. $20 \sin 25°$
2. $42 \tan 62°$
3. $\arcsin 0.8723$
4. $\arctan 2.8703$

Solve for x and round to two decimal places.

5. $\cos 22° = \dfrac{x + 13 \sin 22°}{13 \sin 54°}$
6. $\tan 36° = \dfrac{x + 85 \tan 18°}{85}$

Find the amplitude and period of the function.

7. $f(x) = -4 \sin 2x$
8. $f(x) = \frac{1}{2} \sin \pi x$
9. $g(x) = 3 \cos 3\pi x$
10. $g(x) = 0.2 \cot \dfrac{x}{4}$

1.8 Exercises

In Exercises 1–10, solve the right triangle shown in the figure. (Round to two decimal places.)

1. $A = 20°, \quad b = 10$
2. $B = 54°, \quad c = 15$
3. $B = 71°, \quad b = 24$
4. $A = 8.4°, \quad a = 40.5$
5. $a = 6, \quad b = 10$
6. $a = 25, \quad c = 35$
7. $b = 16, \quad c = 52$
8. $b = 1.32, \quad c = 9.45$
9. $A = 12°15', \quad c = 430.5$
10. $B = 65°12', \quad a = 14.2$

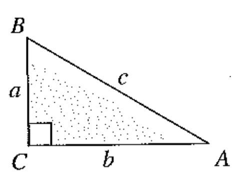


FIGURE FOR 1–10

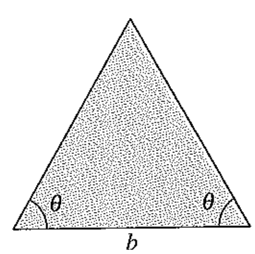
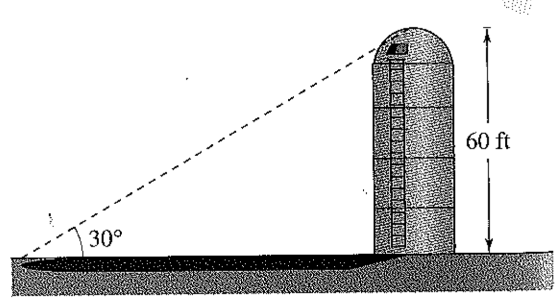


FIGURE FOR 11 AND 12

In Exercises 11 and 12, find the altitude of the isosceles triangle shown in the figure. (Round to two decimal places.)

11. $\theta = 52°, \quad b = 4$ inches
12. $\theta = 18°, \quad b = 10$ meters

13. ***Length of a Shadow*** If the sun is 30° above the horizon, find the length of a shadow cast by a silo that is 60 feet tall (see figure).



Name: ______________ period: ______

14. ***Length of a Shadow*** If the sun is 20° above the horizon, find the length of a shadow cast by a building that is 600 feet tall.

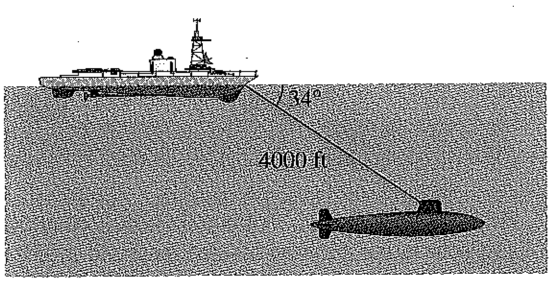
15. ***Height*** A ladder 16 feet long leans against the side of a house. Find the height h from the top of the ladder to the ground if the angle of elevation of the ladder is 74°.

16. ***Height*** The length of a shadow of a tree is 125 feet when the angle of elevation of the sun is 33°. Approximate the height h of the tree.

17. ***Height*** From a point 50 feet in front of a church, the angles of elevation to the base of the steeple and the top of the steeple are 35° and 47° 40′, respectively.
    (a) Draw right triangles that represent the problem. Label the known and unknown quantities.
    (b) Use a trigonometric function to write an equation involving the unknown.
    (c) Find the height of the steeple.

18. ***Height*** From a point 100 feet in front of the public library, the angles of elevation to the base of the flagpole and the top of the pole are 28° and 39° 45′, respectively. The flagpole is mounted on the front of the library's roof. Find the height of the pole.

19. ***Depth of a Submarine*** The sonar of a navy cruiser detects a submarine that is 4000 feet from the cruiser. The angle between the water line and the submarine is 34° (see figure). How deep is the submarine?

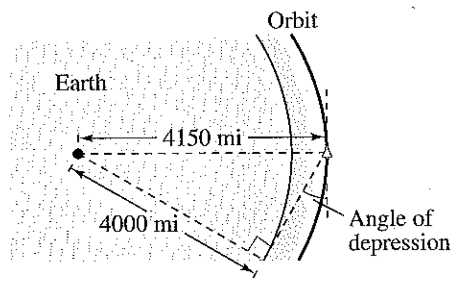


20. ***Height of a Kite*** A 100-foot line is attached to a kite. When the kite has pulled the line taut, the angle of elevation to the kite is approximately 50°. Approximate the height of the kite.

21. ***Angle of Elevation*** An amateur radio operator erects a 75-foot vertical tower for an antenna. Find the angle of elevation to the top of the tower at a point on level ground 50 feet from its base.

22. ***Angle of Elevation*** The height of an outdoor basketball backboard is $12\frac{1}{2}$ feet, and the backboard casts a shadow $17\frac{1}{3}$ feet long.
    (a) Draw a right triangle that represents the problem. Label the known and unknown quantities.
    (b) Use a trigonometric function to write an equation involving the unknown.
    (c) Find the angle of elevation of the sun.

23. ***Angle of Depression*** A spacecraft is traveling in a circular orbit 150 miles above the surface of the earth (see figure). Find the angle of depression from the spacecraft to the horizon. Assume the radius of the earth is 4000 miles.

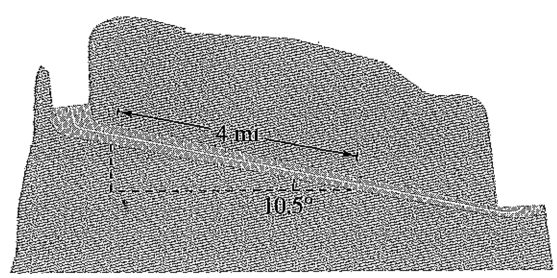


24. ***Angle of Depression*** Find the angle of depression from the top of a lighthouse 250 feet above water level to the water line of a ship 2 miles offshore.

25. ***Airplane Ascent*** When an airplane leaves the runway, its angle of climb is 18° and its speed is 275 feet per second. Find the plane's altitude after one minute.

26. ***Airplane Ascent*** How long will it take the plane in Exercise 25 to climb to an altitude of 10,000 feet?

27. ***Mountain Descent*** A sign on the roadway at the top of a mountain indicates that for the next 4 miles the grade is 10.5° (see figure). Find the change in elevation for a car descending the mountain.

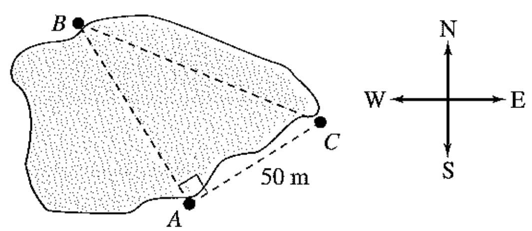


28. ***Mountain Descent*** A sign on the roadway at the top of a mountain indicates that for the next 4 miles the grade is 12%. Find the angle of the grade and the change in elevation for a car descending the mountain.

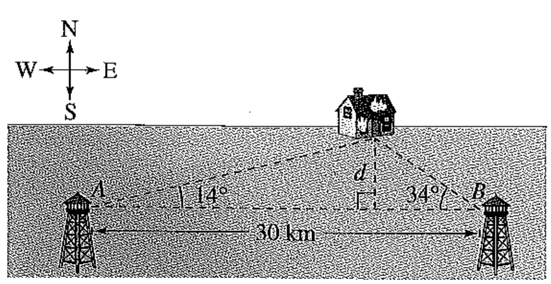
29. ***Navigation*** An airplane flying at 550 miles per hour has a bearing of N 52° E. After flying 1.5 hours, how far north and how far east will the plane have traveled from its point of departure?

30. ***Navigation*** A ship leaves port at noon and has a bearing of S 27° W. If its speed is 20 knots, how many nautical miles south and how many nautical miles west will the ship have traveled by 6:00 P.M.?

31. ***Surveying*** A surveyor wishes to find the distance across a swamp (see figure). The bearing from A to B is N 32° W. The surveyor walks 50 meters from A, and at the point C the bearing to B is N 68° W. Find (a) the bearing from A to C and (b) the distance from A to B.



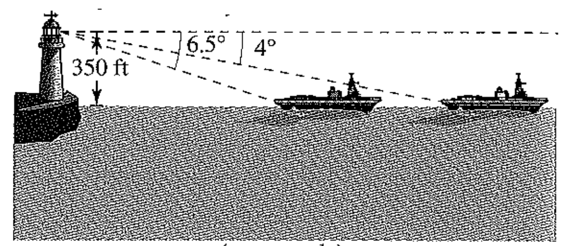
32. ***Location of a Fire*** Two fire towers are 30 kilometers apart, tower A being due west of tower B. A fire is spotted from the towers, and the bearings from A and B are E 14° N and W 34° N, respectively (see figure). Find the distance d of the fire from the line segment AB.



33. ***Navigation*** A ship is 45 miles east and 30 miles south of port. If the captain wants to sail directly to port, what bearing should be taken?

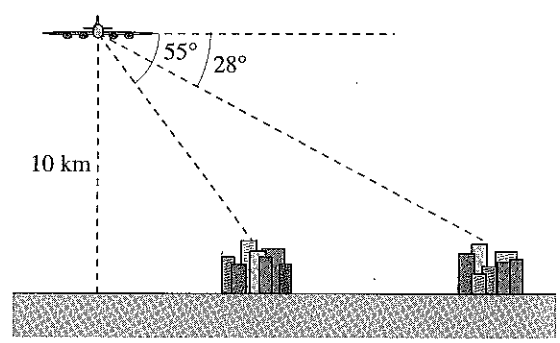
34. ***Navigation*** A plane is 120 miles north and 85 miles east of an airport. If the pilot wants to fly directly to the airport, what bearing should be taken?

35. ***Distance Between Ships*** An observer in a lighthouse 350 feet above sea level observes two ships directly offshore. The angles of depression to the ships are 4° and 6.5° (see figure). How far apart are the ships?



(not to scale)

36. ***Distance Between Towns*** A passenger in an airplane at an altitude of 10 kilometers sees two towns directly to the left of the plane. The angles of depression to the towns are 28° and 55° (see figure). How far apart are the towns?



37. ***Altitude of a Plane*** A plane is observed approaching your home and you assume its speed is 550 miles per hour. If the angle of elevation of the plane is 16° at one time and 57° 1 minute later, approximate the altitude of the plane.

38. ***Height of a Mountain*** While traveling across flat land, you notice a mountain directly in front of you. The angle of elevation to the peak is 3.5°. After you drive 13 miles closer to the mountain, the angle of elevation is 9°. Approximate the height of the mountain.

In Exercises 9–16, sketch a right triangle corresponding to the trigonometric function of the acute angle θ. Use the Pythagorean Theorem to determine the third side, and then find the other five trigonometric functions of θ.

9. $\sin\theta = \frac{2}{3}$ **10.** $\cot\theta = 5$

11. $\sec\theta = 2$ **12.** $\cos\theta = \frac{5}{7}$

13. $\tan\theta = 3$ **14.** $\csc\theta = \frac{17}{4}$

15. $\cot\theta = \frac{3}{2}$ **16.** $\sin\theta = \frac{3}{8}$

In Exercises 17–22, use the given function values and the appropriate trigonometric identities (including the relationship between a trigonometric function and its cofunction of a complementary angle) to find the indicated trigonometric functions.

17. $\sin 60° = \dfrac{\sqrt{3}}{2}$, $\cos 60° = \dfrac{1}{2}$

(a) $\tan 60°$ (b) $\sin 30°$

(c) $\cos 30°$ (d) $\cot 60°$

18. $\sin 30° = \dfrac{1}{2}$, $\tan 30° = \dfrac{\sqrt{3}}{3}$

(a) $\csc 30°$ (b) $\cot 60°$

(c) $\cos 30°$ (d) $\cot 30°$

19. $\csc\theta = 3$, $\sec\theta = \dfrac{3\sqrt{2}}{4}$

(a) $\sin\theta$ (b) $\cos\theta$

(c) $\tan\theta$ (d) $\sec(90° - \theta)$

20. $\sec\theta = 5$, $\tan\theta = 2\sqrt{6}$

(a) $\cos\theta$ (b) $\cot\theta$

(c) $\cot(90° - \theta)$ (d) $\sin\theta$

21. $\cos\alpha = \frac{1}{4}$

(a) $\sec\alpha$ (b) $\sin\alpha$

(c) $\cot\alpha$ (d) $\sin(90° - \alpha)$

22. $\tan\beta = 5$

(a) $\cot\beta$ (b) $\cos\beta$

(c) $\tan(90° - \beta)$ (d) $\csc\beta$

In Exercises 23–32, use trigonometric identities to transform one side of the equation into the other.

23. $\tan\theta\cot\theta = 1$ **24.** $\cos\theta\sec\theta = 1$

25. $\tan\alpha\cos\alpha = \sin\alpha$ **26.** $\cot\alpha\sin\alpha = \cos\alpha$

27. $(1 + \cos\theta)(1 - \cos\theta) = \sin^2\theta$

28. $(1 + \sin\theta)(1 - \sin\theta) = \cos^2\theta$

29. $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

30. $\sin^2\theta - \cos^2\theta = 2\sin^2\theta - 1$

31. $\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \csc\theta\sec\theta$

32. $\dfrac{\tan\beta + \cot\beta}{\tan\beta} = \csc^2\beta$

In Exercises 33–36, evaluate the trigonometric function by memory or by constructing an appropriate triangle for the given special angle.

33. (a) $\cos 60°$ (b) $\tan\dfrac{\pi}{6}$

34. (a) $\csc 30°$ (b) $\sin\dfrac{\pi}{4}$

35. (a) $\cot 45°$ (b) $\cos 45°$

36. (a) $\sin\dfrac{\pi}{3}$ (b) $\csc 45°$

In Exercises 37–46, use a calculator to evaluate each function. Round your answers to four decimal places. (Be sure the calculator is in the correct mode.)

37. (a) $\sin 10°$ (b) $\cos 80°$

38. (a) $\tan 23.5°$ (b) $\cot 66.5°$

39. (a) $\sin 16.35°$ (b) $\csc 16.35°$

40. (a) $\cos 16° 18'$ (b) $\sin 73° 56'$

41. (a) $\sec 42° 12'$ (b) $\csc 48° 7'$

42. (a) $\cos 4° 50' 15''$ (b) $\sec 4° 50' 15''$

43. (a) $\cot\dfrac{\pi}{16}$ (b) $\tan\dfrac{\pi}{16}$

44. (a) $\sec 0.75$ (b) $\cos 0.75$

45. (a) $\csc 1$ (b) $\tan\frac{1}{2}$

46. (a) $\sec\left(\dfrac{\pi}{2} - 1\right)$ (b) $\cot\left(\dfrac{\pi}{2} - \dfrac{1}{2}\right)$

In Exercises 15–18, fill in the blanks. (*Note:* The notation $x \to c^+$ indicates that x approaches c from the right and $x \to c^-$ indicates that x approaches c from the left.)

15. As $x \to \frac{\pi}{2}^-$, $\sin x \to$ ____ and $\csc x \to$ ____.

16. As $x \to 0^+$, $\cos x \to$ ____ and $\sec x \to$ ____.

17. As $x \to \frac{\pi}{2}^-$, $\tan x \to$ ____ and $\cot x \to$ ____.

18. As $x \to \pi^+$, $\sin x \to$ ____ and $\csc x \to$ ____.

In Exercises 19–24, match the trigonometric expression with one of the following.

(a) -1 **(b)** $\cos x$ **(c)** $\cot x$
(d) 1 **(e)** $-\tan x$ **(f)** $\sin x$

19. $\sec x \cos x$
20. $\cot x \sin x$
21. $\tan^2 x - \sec^2 x$
22. $(1 - \cos^2 x)(\csc x)$
23. $\dfrac{\sin(-x)}{\cos(-x)}$
24. $\dfrac{\sin[(\pi/2) - x]}{\cos[(\pi/2) - x]}$

In Exercises 25–30, match the trigonometric expression with one of the following.

(a) $\csc x$ **(b)** $\tan x$ **(c)** $\sin^2 x$
(d) $\sin x \tan x$ **(e)** $\sec^2 x$ **(f)** $\sec^2 x + \tan^2 x$

25. $\sin x \sec x$
26. $\cos^2 x(\sec^2 x - 1)$
27. $\sec^4 x - \tan^4 x$
28. $\cot x \sec x$
29. $\dfrac{\sec^2 x - 1}{\sin^2 x}$
30. $\dfrac{\cos^2[(\pi/2) - x]}{\cos x}$

In Exercises 31–44, use the fundamental identities to simplify the expression.

31. $\tan \phi \csc \phi$
32. $\sin \phi(\csc \phi - \sin \phi)$
33. $\cos \beta \tan \beta$
34. $\sec^2 x(1 - \sin^2 x)$
35. $\dfrac{\cot x}{\csc x}$
36. $\dfrac{\csc \theta}{\sec \theta}$
37. $\sec \alpha \cdot \dfrac{\sin \alpha}{\tan \alpha}$
38. $\dfrac{1}{\tan^2 x + 1}$
39. $\dfrac{\sin(-x)}{\cos x}$
40. $\dfrac{\tan^2 \theta}{\sec^2 \theta}$
41. $\cos\left(\dfrac{\pi}{2} - x\right)\sec x$
42. $\cot\left(\dfrac{\pi}{2} - x\right)\cos x$
43. $\dfrac{\cos^2 y}{1 - \sin y}$
44. $\cos t(1 + \tan^2 t)$

In Exercises 45–52, factor the expression and use the fundamental identities to simplify.

45. $\tan^2 x - \tan^2 x \sin^2 x$
46. $\sec^2 x \tan^2 x + \sec^2 x$
47. $\sin^2 x \sec^2 x - \sin^2 x$
48. $\dfrac{\sec^2 x - 1}{\sec x - 1}$
49. $\tan^4 x + 2 \tan^2 x + 1$
50. $1 - 2 \cos^2 x + \cos^4 x$
51. $\sin^4 x - \cos^4 x$
52. $\csc^3 x - \csc^2 x - \csc x + 1$

In Exercises 53–56, perform the multiplication and use the fundamental identities to simplify.

53. $(\sin x + \cos x)^2$
54. $(\cot x + \csc x)(\cot x - \csc x)$
55. $(\sec x + 1)(\sec x - 1)$
56. $(3 - 3 \sin x)(3 + 3 \sin x)$

In Exercises 57–60, perform the addition or subtraction and use the fundamental identities to simplify.

57. $\dfrac{1}{1 + \cos x} + \dfrac{1}{1 - \cos x}$
58. $\dfrac{1}{\sec x + 1} - \dfrac{1}{\sec x - 1}$
59. $\dfrac{\cos x}{1 + \sin x} + \dfrac{1 + \sin x}{\cos x}$
60. $\tan x - \dfrac{\sec^2 x}{\tan x}$

In Exercises 61–64, rewrite the expression so that it is *not* in fractional form.

61. $\dfrac{\sin^2 y}{1 - \cos y}$ **62.** $\dfrac{5}{\tan x + \sec x}$

63. $\dfrac{3}{\sec x - \tan x}$ **64.** $\dfrac{\tan^2 x}{\csc x + 1}$

Numerical and Graphical Analysis **In Exercises 65–68, use a graphing utility to complete the table and graph the functions. Make a conjecture about y_1 and y_2.**

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1							
y_2							

65. $y_1 = \cos\left(\dfrac{\pi}{2} - x\right), \quad y_2 = \sin x$

66. $y_1 = \cos x + \sin x \tan x, \quad y_2 = \sec x$

67. $y_1 = \dfrac{\cos x}{1 - \sin x}, \quad y_2 = \dfrac{1 + \sin x}{\cos x}$

68. $y_1 = \sec^4 x - \sec^2 x, \quad y_2 = \tan^2 x + \tan^4 x$

In Exercises 69 and 70, use a graphing utility to determine which of the six trigonometric functions is equal to the expression.

69. $\cos x \cot x + \sin x$

70. $\dfrac{1}{2}\left(\dfrac{1 + \sin \theta}{\cos \theta} + \dfrac{\cos \theta}{1 + \sin \theta}\right)$

In Exercises 71–76, use the trigonometric substitution to write the algebraic expression as a trigonometric function of θ, where $0 < \theta < \pi/2$.

71. $\sqrt{25 - x^2}, \quad x = 5 \sin \theta$

72. $\sqrt{16 - 4x^2}, \quad x = 2 \sin \theta$

73. $\sqrt{x^2 - 9}, \quad x = 3 \sec \theta$

74. $\sqrt{x^2 - 4}, \quad x = 2 \sec \theta$

75. $\sqrt{x^2 + 25}, \quad x = 5 \tan \theta$

76. $\sqrt{x^2 + 100}, \quad x = 10 \tan \theta$

In Exercises 77–80, use a graphing utility to solve the equation for θ, where $0 \le \theta < 2\pi$.

77. $\sin \theta = \sqrt{1 - \cos^2 \theta}$ **78.** $\cos \theta = -\sqrt{1 - \sin^2 \theta}$

79. $\sec \theta = \sqrt{1 + \tan^2 \theta}$ **80.** $\tan \theta = \sqrt{\sec^2 \theta - 1}$

In Exercises 81 and 82, rewrite the expression as a single logarithm and simplify the result.

81. $\ln|\cos \theta| - \ln|\sin \theta|$

82. $\ln|\cot t| + \ln(1 + \tan^2 t)$

In Exercises 83–86, determine whether or not the equation is an identity, and give a reason for your answer.

83. $(\sin k\theta)/(\cos k\theta) = \tan \theta$, k is a constant.

84. $1/(5 \cos \theta) = 5 \sec \theta$

85. $\sin \theta \csc \theta = 1$

86. $\sin \theta \csc \phi = 1$

In Exercises 87–90, use a calculator to demonstrate the identity for the given values of θ.

87. $\csc^2 \theta - \cot^2 \theta = 1$, (a) $\theta = 132°$, (b) $\theta = \dfrac{2\pi}{7}$

88. $\tan^2 \theta + 1 = \sec^2 \theta$, (a) $\theta = 346°$, (b) $\theta = 3.1$

89. $\cos\left(\dfrac{\pi}{2} - \theta\right) = \sin \theta$, (a) $\theta = 80°$, (b) $\theta = 0.8$

90. $\sin(-\theta) = -\sin \theta$, (a) $\theta = 250°$, (b) $\theta = \dfrac{1}{2}$

91. Express each of the other trigonometric functions of θ in terms of $\sin \theta$.

92. Express each of the other trigonometric functions of θ in terms of $\cos \theta$.

Review **Solve Exercises 93–96 as a review of the skills and problem-solving techniques you learned in previous sections. Perform the operations and simplify.**

93. $(\sqrt{x} + 5)(\sqrt{x} - 5)$ **94.** $\sqrt{v}(\sqrt{20} - \sqrt{5})$

95. $(2\sqrt{z} + 3)^2$ **96.** $50x/(\sqrt{30} - 5)$

WARM UP

Factor each expression and, if possible, simplify the result.

1. (a) $x^2 - x^2y^2$
   (b) $\sin^2 x - \sin^2 x \cos^2 x$
2. (a) $x^2 + x^2y^2$
   (b) $\cos^2 x + \cos^2 x \tan^2 x$
3. (a) $x^4 - 1$
   (b) $\tan^4 x - 1$
4. (a) $z^3 + 1$
   (b) $\tan^3 x + 1$
5. (a) $x^3 - x^2 + x - 1$
   (b) $\cot^3 x - \cot^2 x + \cot x - 1$
6. (a) $x^4 - 2x^2 + 1$
   (b) $\sin^4 x - 2\sin^2 x + 1$

Perform the operations and, if possible, simplify the result.

7. (a) $\dfrac{y^2}{x} - x$
   (b) $\dfrac{\csc^2 x}{\cot x} - \cot x$
8. (a) $1 - \dfrac{1}{x^2}$
   (b) $1 - \dfrac{1}{\sec^2 x}$
9. (a) $\dfrac{y}{1+z} + \dfrac{1+z}{y}$
   (b) $\dfrac{\sin x}{1+\cos x} + \dfrac{1+\cos x}{\sin x}$
10. (a) $\dfrac{y}{z} - \dfrac{z}{1+y}$
    (b) $\dfrac{\tan x}{\sec x} - \dfrac{\sec x}{1+\tan x}$

2.2 Exercises

In Exercises 1–44, verify the identity.

1. $\sin t \csc t = 1$
2. $\tan y \cot y = 1$
3. $(1 + \sin\alpha)(1 - \sin\alpha) = \cos^2\alpha$
4. $\cot^2 y(\sec^2 y - 1) = 1$
5. $\cos^2\beta - \sin^2\beta = 1 - 2\sin^2\beta$
6. $\cos^2\beta - \sin^2\beta = 2\cos^2\beta - 1$
7. $\tan^2\theta + 4 = \sec^2\theta + 3$
8. $2 - \sec^2 z = 1 - \tan^2 z$
9. $\sin^2\alpha - \sin^4\alpha = \cos^2\alpha - \cos^4\alpha$
10. $\cos x + \sin x \tan x = \sec x$
11. $\dfrac{\sec^2 x}{\tan x} = \sec x \csc x$
12. $\dfrac{\cot^3 t}{\csc t} = \cos t(\csc^2 t - 1)$
13. $\dfrac{\cot^2 t}{\csc t} = \csc t - \sin t$
14. $\dfrac{1}{\sin x} - \sin x = \dfrac{\cos^2 x}{\sin x}$
15. $\sin^{1/2} x \cos x - \sin^{5/2} x \cos x = \cos^3 x\sqrt{\sin x}$
16. $\sec^6 x(\sec x \tan x) - \sec^4 x(\sec x \tan x) = \sec^5 x \tan^3 x$
17. $\dfrac{1}{\sec x \tan x} = \csc x - \sin x$

18. $\dfrac{\sec\theta - 1}{1 - \cos\theta} = \sec\theta$

19. $\csc x - \sin x = \cos x \cot x$

20. $\sec x - \cos x = \sin x \tan x$

21. $\cos x + \sin x \tan x = \sec x$

22. $\dfrac{\sec x + \tan x}{\sec x - \tan x} = (\sec x + \tan x)^2$

23. $\dfrac{1}{\tan x} + \dfrac{1}{\cot x} = \tan x + \cot x$

24. $\dfrac{1}{\sin x} - \dfrac{1}{\csc x} = \csc x - \sin x$

25. $\dfrac{\cos\theta \cot\theta}{1 - \sin\theta} - 1 = \csc\theta$

26. $\dfrac{1 + \sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1 + \sin\theta} = 2\sec\theta$

27. $\dfrac{1}{\cot x + 1} + \dfrac{1}{\tan x + 1} = 1$

28. $\cos x - \dfrac{\cos x}{1 - \tan x} = \dfrac{\sin x \cos x}{\sin x - \cos x}$

29. $\cos\left(\dfrac{\pi}{2} - x\right)\csc x = 1$

30. $\dfrac{\cos[(\pi/2) - x]}{\sin[(\pi/2) - x]} = \tan x$

31. $\dfrac{\csc(-x)}{\sec(-x)} = -\cot x$

32. $(1 + \sin y)[1 + \sin(-y)] = \cos^2 y$

33. $\dfrac{\cos(-\theta)}{1 + \sin(-\theta)} = \sec\theta + \tan\theta$

34. $\dfrac{1 + \sec(-\theta)}{\sin(-\theta) + \tan(-\theta)} = -\csc\theta$

35. $\dfrac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y} = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

36. $\dfrac{\tan x + \tan y}{1 - \tan x \tan y} = \dfrac{\cot x + \cot y}{\cot x \cot y - 1}$

37. $\dfrac{\tan x + \cot y}{\tan x \cot y} = \tan y + \cot x$

38. $\dfrac{\cos x - \cos y}{\sin x + \sin y} + \dfrac{\sin x - \sin y}{\cos x + \cos y} = 0$

39. $\sqrt{\dfrac{1 + \sin\theta}{1 - \sin\theta}} = \dfrac{1 + \sin\theta}{|\cos\theta|}$

40. $\sqrt{\dfrac{1 - \cos\theta}{1 + \cos\theta}} = \dfrac{1 - \cos\theta}{|\sin\theta|}$

41. $\sin^2 x + \sin^2\left(\dfrac{\pi}{2} - x\right) = 1$

42. $\sec^2 y - \cot^2\left(\dfrac{\pi}{2} - y\right) = 1$

43. $\csc x \cos\left(\dfrac{\pi}{2} - x\right) = 1$

44. $\sec^2\left(\dfrac{\pi}{2} - x\right) - 1 = \cot^2 x$

In Exercises 45–56, verify the identity algebraically, and use a graphing utility to confirm it graphically.

45. $2\sec^2 x - 2\sec^2 x \sin^2 x - \sin^2 x - \cos^2 x = 1$

46. $\csc x(\csc x - \sin x) + \dfrac{\sin x - \cos x}{\sin x} + \cot x = \csc^2 x$

47. $2 + \cos^2 x - 3\cos^4 x = \sin^2 x(2 + 3\cos^2 x)$

48. $4\tan^4 x + \tan^2 x - 3 = \sec^2 x(4\tan^2 x - 3)$

49. $\csc^4 x - 2\csc^2 x + 1 = \cot^4 x$

50. $\sin x(1 - 2\cos^2 x + \cos^4 x) = \sin^5 x$

51. $\sec^4\theta - \tan^4\theta = 1 + 2\tan^2\theta$

52. $\csc^4\theta - \cot^4\theta = 2\csc^2\theta - 1$

53. $\dfrac{\sin\beta}{1 - \cos\beta} = \dfrac{1 + \cos\beta}{\sin\beta}$

54. $\dfrac{\cot\alpha}{\csc\alpha - 1} = \dfrac{\csc\alpha + 1}{\cot\alpha}$

55. $\dfrac{\tan^3\alpha - 1}{\tan\alpha - 1} = \tan^2\alpha + \tan\alpha + 1$

56. $\dfrac{\sin^3\beta + \cos^3\beta}{\sin\beta + \cos\beta} = 1 - \sin\beta\cos\beta$

In Exercises 21–30, write the expression as the sine, cosine, or tangent of an angle.

21. $\cos 25° \cos 15° - \sin 25° \sin 15°$

22. $\sin 140° \cos 50° + \cos 140° \sin 50°$

23. $\sin 230° \cos 30° - \cos 230° \sin 30°$

24. $\cos 20° \cos 30° + \sin 20° \sin 30°$

25. $\dfrac{\tan 325° - \tan 86°}{1 + \tan 325° \tan 86°}$

26. $\dfrac{\tan 140° - \tan 60°}{1 + \tan 140° \tan 60°}$

27. $\sin 3 \cos 1.2 - \cos 3 \sin 1.2$

28. $\cos \dfrac{\pi}{7} \cos \dfrac{\pi}{5} - \sin \dfrac{\pi}{7} \sin \dfrac{\pi}{5}$

29. $\dfrac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$

30. $\cos 3x \cos 2y + \sin 3x \sin 2y$

In Exercises 31–38, find the exact value of the trigonometric function given that $\sin u = \frac{5}{13}$ and $\cos v = -\frac{3}{5}$. (Both u and v are in Quadrant II.)

31. $\sin(u + v)$

32. $\cos(v - u)$

33. $\cos(u + v)$

34. $\sin(u - v)$

35. $\sec(u + v)$

36. $\csc(u - v)$

37. $\tan(u - v)$

38. $\cot(u + v)$

In Exercises 39–44, find the exact value of the trigonometric function given that $\sin u = -\frac{7}{25}$ and $\cos v = -\frac{4}{5}$. (Both u and v are in Quadrant III.)

39. $\cos(u + v)$

40. $\sin(u + v)$

41. $\sin(v - u)$

42. $\cos(u - v)$

43. $\csc(u + v)$

44. $\sec(v - u)$

In Exercises 45–58, verify the identity.

45. $\sin(3\pi - x) = \sin x$

46. $\sin\left(\dfrac{\pi}{2} + x\right) = \cos x$

47. $\sin\left(\dfrac{\pi}{6} + x\right) = \dfrac{1}{2}(\cos x + \sqrt{3} \sin x)$

48. $\cos\left(\dfrac{5\pi}{4} - x\right) = -\dfrac{\sqrt{2}}{2}(\cos x + \sin x)$

49. $\cos(\pi - \theta) + \sin\left(\dfrac{\pi}{2} + \theta\right) = 0$

50. $\tan\left(\dfrac{\pi}{4} - \theta\right) = \dfrac{1 - \tan \theta}{1 + \tan \theta}$

51. $\cos(x + y) \cos(x - y) = \cos^2 x - \sin^2 y$

52. $\sin(x + y) \sin(x - y) = \sin^2 x - \sin^2 y$

53. $\sin(x + y) + \sin(x - y) = 2 \sin x \cos y$

54. $\cos(x + y) + \cos(x - y) = 2 \cos x \cos y$

55. $\cos(n\pi + \theta) = (-1)^n \cos \theta$, n is an integer.

56. $\sin(n\pi + \theta) = (-1)^n \sin \theta$, n is an integer.

57. $a \sin B\theta + b \cos B\theta = \sqrt{a^2 + b^2} \sin(B\theta + C)$, where $C = \arctan(b/a)$, $a > 0$

58. $a \sin B\theta + b \cos B\theta = \sqrt{a^2 + b^2} \cos(B\theta - C)$, where $C = \arctan(a/b)$, $b > 0$

In Exercises 59–62, verify the identity algebraically and use a graphing utility to confirm it graphically.

59. $\cos\left(\dfrac{3\pi}{2} - x\right) = -\sin x$

60. $\cos(\pi + x) = -\cos x$

61. $\sin\left(\dfrac{3\pi}{2} + \theta\right) + \sin(\pi - \theta) = \sin \theta - \cos \theta$

62. $\tan(\pi + \theta) = \tan \theta$

In Exercises 63–66, use the formulas given in Exercises 57 and 58 to write the trigonometric expression in the following forms.

(a) $\sqrt{a^2 + b^2} \sin(B\theta + C)$

(b) $\sqrt{a^2 + b^2} \cos(B\theta - C)$

63. $\sin \theta + \cos \theta$

64. $3 \sin 2\theta + 4 \cos 2\theta$

65. $12 \sin 3\theta + 5 \cos 3\theta$

66. $\sin 2\theta - \cos 2\theta$

In Exercises 67 and 68, use the formulas given in Exercises 57 and 58 to write the trigonometric expression in the form $a \sin B\theta + b \cos B\theta$.

67. $2 \sin\left(\theta + \dfrac{\pi}{2}\right)$

68. $5 \cos\left(\theta + \dfrac{3\pi}{4}\right)$

WARM UP

Factor the trigonometric expression.

1. $2 \sin x + \sin x \cos x$
2. $\cos^2 x - \cos x - 2$

Find all solutions of the equation in the interval $[0, 2\pi)$.

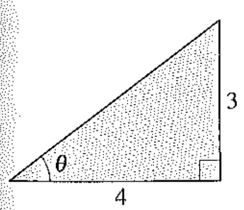
3. $\sin 2x = 0$
4. $\cos 2x = 0$
5. $\cos \frac{x}{2} = 0$
6. $\sin \frac{x}{2} = 0$

Simplify the expression.

7. $\frac{1 - \cos(\pi/4)}{2}$
8. $\frac{1 + \cos(\pi/3)}{2}$
9. $\frac{2 \sin 3x \cos x}{2 \cos 3x \cos x}$
10. $\frac{(1 - 2 \sin^2 x) \cos x}{2 \sin^2 x \cos x}$

2.5 Exercises

In Exercises 1–8, use the figure to find the exact value of the trigonometric function.



1. $\sin \theta$
2. $\tan \theta$
3. $\cos 2\theta$
4. $\sin 2\theta$
5. $\tan 2\theta$
6. $\sec 2\theta$
7. $\csc 2\theta$
8. $\cot 2\theta$

In Exercises 9–18, find the exact solutions of the equation in the interval $[0, 2\pi)$.

9. $\sin 2x - \sin x = 0$
10. $\sin 2x + \cos x = 0$
11. $4 \sin x \cos x = 1$
12. $\sin 2x \sin x = \cos x$
13. $\cos 2x - \cos x = 0$
14. $\cos 2x + \sin x = 0$
15. $\tan 2x - \cot x = 0$
16. $\tan 2x - 2 \cos x = 0$
17. $\sin 4x = -2 \sin 2x$
18. $(\sin 2x + \cos 2x)^2 = 1$

In Exercises 19–22, use a double-angle formula to rewrite the expression.

19. $6 \sin x \cos x$
20. $4 \sin x \cos x + 2$
21. $4 - 8 \sin^2 x$
22. $(\cos x + \sin x)(\cos x - \sin x)$

In Exercises 23–28, find the exact values of $\sin 2u$, $\cos 2u$, and $\tan 2u$ using the double-angle formulas.

23. $\sin u = \frac{3}{5}, \quad 0 < u < \frac{\pi}{2}$
24. $\cos u = -\frac{2}{3}, \quad \frac{\pi}{2} < u < \pi$

25. $\tan u = \frac{1}{2}, \quad \pi < u < \frac{3\pi}{2}$

26. $\cot u = -4, \quad \frac{3\pi}{2} < u < 2\pi$

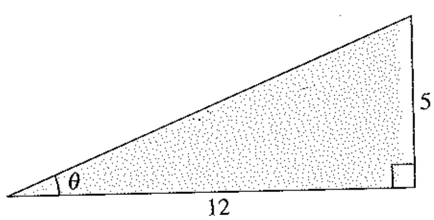
27. $\sec u = -\frac{5}{2}, \quad \frac{\pi}{2} < u < \pi$

28. $\csc u = 3, \quad \frac{\pi}{2} < u < \pi$

In Exercises 29–34, use the power-reducing formulas to rewrite the expression in terms of the first power of the cosine.

29. $\cos^4 x$
30. $\sin^4 x$
31. $\sin^2 x \cos^2 x$
32. $\cos^2 x$
33. $\sin^2 x \cos^4 x$
34. $\sin^4 x \cos^2 x$

In Exercises 35–40, use the figure to find the exact value of the trigonometric function.



35. $\cos \frac{\theta}{2}$
36. $\sin \frac{\theta}{2}$
37. $\tan \frac{\theta}{2}$
38. $2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$
39. $\csc \frac{\theta}{2}$
40. $\cot \frac{\theta}{2}$

In Exercises 41–46, use the half-angle formulas to determine the exact values of the sine, cosine, and tangent of the angle.

41. $105°$
42. $165°$
43. $112° 30'$
44. $67° 30'$
45. $\frac{\pi}{8}$
46. $\frac{\pi}{12}$

In Exercises 47–52, find the exact values of $\sin(u/2)$, $\cos(u/2)$, and $\tan(u/2)$ using the half-angle formulas.

47. $\sin u = \frac{5}{13}, \quad \frac{\pi}{2} < u < \pi$

48. $\cos u = \frac{3}{5}, \quad 0 < u < \frac{\pi}{2}$

49. $\tan u = -\frac{5}{8}, \quad \frac{3\pi}{2} < u < 2\pi$

50. $\cot u = 3, \quad \pi < u < \frac{3\pi}{2}$

51. $\csc u = -\frac{5}{3}, \quad \pi < u < \frac{3\pi}{2}$

52. $\sec u = -\frac{7}{2}, \quad \frac{\pi}{2} < u < \pi$

In Exercises 53–56, use the half-angle formulas to simplify the expression.

53. $\sqrt{\frac{1 - \cos 6x}{2}}$
54. $\sqrt{\frac{1 + \cos 4x}{2}}$
55. $-\sqrt{\frac{1 - \cos 8x}{1 + \cos 8x}}$
56. $-\sqrt{\frac{1 - \cos(x - 1)}{2}}$

In Exercises 57–60, find the exact zeros of the function in the interval $[0, 2\pi)$. Use the graphing utility to graph the function and verify the zeros.

57. $f(x) = \sin \frac{x}{2} + \cos x$

58. $h(x) = \sin \frac{x}{2} + \cos x - 1$

59. $h(x) = \cos \frac{x}{2} - \sin x$

60. $g(x) = \tan \frac{x}{2} - \sin x$

In Exercises 61–70, use the product-to-sum formulas to write the product as a sum or difference.

61. $6 \sin \frac{\pi}{4} \cos \frac{\pi}{4}$
62. $4 \sin \frac{\pi}{3} \cos \frac{5\pi}{6}$
63. $\sin 5\theta \cos 3\theta$
64. $3 \sin 2\alpha \sin 3\alpha$
65. $5 \cos(-5\beta) \cos 3\beta$
66. $\cos 2\theta \cos 4\theta$

67. $\sin(x + y)\sin(x - y)$
68. $\sin(x + y)\cos(x - y)$
69. $\sin(\theta + \pi)\cos(\theta - \pi)$
70. $10 \cos 75^\circ \cos 15^\circ$

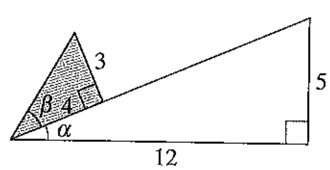
In Exercises 71–80, use the sum-to-product formulas to write the sum or difference as a product.

71. $\sin 60^\circ + \sin 30^\circ$
72. $\cos 120^\circ + \cos 30^\circ$
73. $\cos \dfrac{3\pi}{4} - \cos \dfrac{\pi}{4}$
74. $\sin 5\theta - \sin 3\theta$
75. $\cos 6x + \cos 2x$
76. $\sin x + \sin 5x$
77. $\sin(\alpha + \beta) - \sin(\alpha - \beta)$
78. $\cos\left(\theta + \dfrac{\pi}{2}\right) - \cos\left(\theta - \dfrac{\pi}{2}\right)$
79. $\cos(\phi + 2\pi) + \cos \phi$
80. $\sin\left(x + \dfrac{\pi}{2}\right) + \sin\left(x - \dfrac{\pi}{2}\right)$

In Exercises 81–84, find the exact zeros of the function in the interval $[0, 2\pi)$. Use a graphing utility to graph the function and verify the zeros.

81. $g(x) = \sin 6x + \sin 2x$
82. $h(x) = \cos 2x - \cos 6x$
83. $f(x) = \dfrac{\cos 2x}{\sin 3x - \sin x} - 1$
84. $f(x) = \sin^2 3x - \sin^2 x$

In Exercises 85–88, use the figure to find the exact value of the trigonometric function in two ways.



85. $\sin^2 \alpha$
86. $\cos^2 \alpha$
87. $\sin \alpha \cos \beta$
88. $\cos \alpha \sin \beta$

In Exercises 89–102, verify the identity.

89. $\csc 2\theta = \dfrac{\csc \theta}{2 \cos \theta}$
90. $\sec 2\theta = \dfrac{\sec^2 \theta}{2 - \sec^2 \theta}$
91. $\cos^2 2\alpha - \sin^2 2\alpha = \cos 4\alpha$
92. $\cos^4 x - \sin^4 x = \cos 2x$
93. $(\sin x + \cos x)^2 = 1 + \sin 2x$
94. $\sin \dfrac{\alpha}{3} \cos \dfrac{\alpha}{3} = \dfrac{1}{2} \sin \dfrac{2\alpha}{3}$
95. $1 + \cos 10y = 2 \cos^2 5y$
96. $\dfrac{\cos 3\beta}{\cos \beta} = 1 - 4 \sin^2 \beta$
97. $\sec \dfrac{u}{2} = \pm\sqrt{\dfrac{2 \tan u}{\tan u + \sin u}}$
98. $\tan \dfrac{u}{2} = \csc u - \cot u$
99. $\dfrac{\cos 4x + \cos 2x}{\sin 4x + \sin 2x} = \cot 3x$
100. $\dfrac{\sin x \pm \sin y}{\sin x + \cos y} = \tan \dfrac{x \pm y}{2}$
101. $\dfrac{\cos t + \cos 3t}{\sin 3t - \sin t} = \cot t$
102. $\sin\left(\dfrac{\pi}{6} + x\right) + \sin\left(\dfrac{\pi}{6} - x\right) = \cos x$

In Exercises 103–106, verify the identity algebraically. Use a graphing utility to confirm the identity.

103. $\cos 3\beta = \cos^3 \beta - 3 \sin^2 \beta \cos \beta$
104. $\sin 4\beta = 4 \sin \beta \cos \beta(1 - 2 \sin^2 \beta)$
105. $(\cos 4x - \cos 2x)/(2 \sin 3x) = -\sin x$
106. $(\cos 3x - \cos x)/(\sin 3x - \sin x) = -\tan 2x$

In Exercises 107 and 108, graph the function by using the power-reducing formulas.

107. $f(x) = \sin^2 x$
108. $f(x) = \cos^2 x$

In Exercises 11–24, solve the equation.

11. $2\cos x + 1 = 0$

12. $2\sin x - 1 = 0$

13. $\sqrt{3}\csc x - 2 = 0$

14. $\tan x + 1 = 0$

15. $3\sec^2 x - 4 = 0$

16. $\csc^2 x - 2 = 0$

17. $2\sin^2 2x = 1$

18. $\tan^2 3x = 3$

19. $4\sin^2 x - 3 = 0$

20. $\sin x(\sin x + 1) = 0$

21. $\sin^2 x = 3\cos^2 x$

22. $\tan 3x(\tan x - 1) = 0$

23. $(3\tan^2 x - 1)(\tan^2 x - 3) = 0$

24. $\cos 2x(2\cos x + 1) = 0$

In Exercises 25–40, find all solutions of the equation in the interval $[0, 2\pi)$.

25. $\cos^3 x = \cos x$

26. $\tan^2 x - 1 = 0$

27. $3\tan^3 x = \tan x$

28. $2\sin^2 x = 2 + \cos x$

29. $\sec^2 x - \sec x = 2$

30. $\sec x \csc x = 2\csc x$

31. $2\sin x + \csc x = 0$

32. $\sin 2x = -\dfrac{\sqrt{3}}{2}$

33. $\csc x + \cot x = 1$

34. $\tan 3x = 1$

35. $\cos\dfrac{x}{2} = \dfrac{\sqrt{2}}{2}$

36. $\sec 4x = 2$

37. $\dfrac{1 + \cos x}{1 - \cos x} = 0$

38. $2\sin^2 x + 3\sin x + 1 = 0$

39. $2\sec^2 x + \tan^2 x - 3 = 0$

40. $\cos x + \sin x \tan x = 2$

In Exercises 41 and 42, solve both equations. How do the solutions of the algebraic equation compare to the solutions of the trigonometric equation?

41. $6y^2 - 13y + 6 = 0$
$6\cos^2 x - 13\cos x + 6 = 0$

42. $y^2 + y - 20 = 0$
$\sin^2 x + \sin x - 20 = 0$

In Exercises 43–56, use a graphing utility to approximate the solutions of the equation in the interval $[0, 2\pi)$.

43. $2\cos x - \sin x = 0$

44. $4\sin^3 x + 2\sin^2 x - 2\sin x - 1 = 0$

45. $\dfrac{1 + \sin x}{\cos x} + \dfrac{\cos x}{1 + \sin x} = 4$

46. $\dfrac{\cos x \cot x}{1 - \sin x} = 3$

47. $2\sin x - x = 0$

48. $x\cos x - 1 = 0$

49. $\sec^2 x + 0.5\tan x - 1 = 0$

50. $\csc^2 x + 0.5\cot x - 5 = 0$

51. $2\tan^2 x + 7\tan x - 15 = 0$

52. $12\cos^2 x + 5\cos x - 3 = 0$

53. $12\sin^2 x - 13\sin x + 3 = 0$

54. $3\tan^2 x + 4\tan x - 4 = 0$

55. $\sin^2 x + 2\sin x - 1 = 0$

56. $4\cos^2 x - 4\cos x - 1 = 0$

In Exercises 57 and 58, (a) use a graphing utility to graph the function and approximate the maximum and minimum points on the graph in the interval $[0, 2\pi)$, and (b) solve the trigonometric equation and demonstrate that its solutions are the x-coordinates of the maximum and minimum points of f. (Calculus is required to find the trigonometric equation.)

	Function	*Trigonometric Equation*
57.	$f(x) = \sin x + \cos x$	$\cos x - \sin x = 0$
58.	$f(x) = 2\sin x + \cos 2x$	$2\cos x - 4\sin x \cos x = 0$

Fixed Point **In Exercises 59 and 60, find the smallest positive fixed point of the function f. [A fixed point of a function f is a real number c such that $f(c) = c$.]**

59. $f(x) = \tan\dfrac{\pi x}{4}$

60. $f(x) = \cos x$

WARM UP

Solve for x.

1. $x \ln 2 = \ln 3$
2. $(x - 1) \ln 4 = 2$
3. $2xe^2 = e^3$
4. $4xe^{-1} = 8$
5. $x^2 - 4x + 5 = 0$
6. $2x^2 - 3x + 1 = 0$

Simplify the expression.

7. $\log_{10} 100^x$
8. $\log_4 64^x$
9. $\ln e^{2x}$
10. $\ln e^{-x^2}$

5.4 Exercises

In Exercises 1–6, determine whether the x-values are solutions of the equation.

1. $4^{2x-7} = 64$
(a) $x = 5$
(b) $x = 2$

2. $2^{3x+1} = 32$
(a) $x = -1$
(b) $x = 2$

3. $3e^{x+2} = 75$
(a) $x = -2 + e^{25}$
(b) $x = -2 + \ln 25$
(c) $x \approx 1.2189$

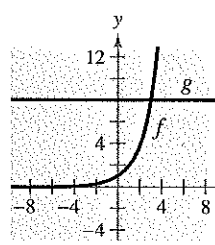
4. $5^{2x+3} = 812$
(a) $x = -1.5 + \log_5 \sqrt{812}$
(b) $x \approx 0.5813$
(c) $x = \dfrac{1}{2}\left(-3 + \dfrac{\ln 812}{\ln 5}\right)$

5. $\log_4(3x) = 3$
(a) $x \approx 20.3560$
(b) $x = -4$
(c) $x = \frac{64}{3}$

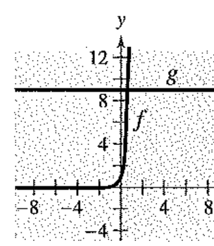
6. $\ln(x - 1) = 3.8$
(a) $x = 1 + e^{3.8}$
(b) $x \approx 45.7012$
(c) $x = 1 + \ln 3.8$

In Exercises 7–10, approximate the point of intersection of the graphs of f and g. Then solve the equation $f(x) = g(x)$ algebraically.

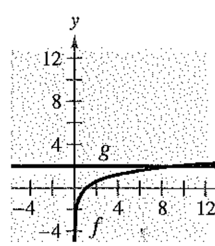
7. $f(x) = 2^x$
$g(x) = 8$



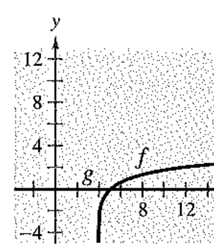
8. $f(x) = 27^x$
$g(x) = 9$



9. $f(x) = \log_3 x$
$g(x) = 2$



10. $f(x) = \ln(x - 4)$
$g(x) = 0$



In Exercises 11–20, solve for x.

11. $4^x = 16$
12. $3^x = 243$
13. $7^x = \frac{1}{49}$
14. $8^x = 4$
15. $\left(\frac{3}{4}\right)^x = \frac{27}{64}$
16. $3^{x-1} = 27$
17. $\log_4 x = 3$
18. $\log_x 625 = 4$
19. $\log_{10} x = -1$
20. $\ln(2x - 1) = 0$

In Exercises 21–26, apply the inverse properties of $\ln x$ and e^x to simplify the expression.

21. $\log_{10} 10^{x^2}$
22. $\log_6 6^{2x-1}$
23. $e^{\ln(5x+2)}$
24. $-1 + \ln e^{2x}$
25. $e^{\ln x^2}$
26. $-8 + e^{\ln x^3}$

In Exercises 27–46, solve the exponential equation algebraically. Round the result to three decimal places.

27. $e^x = 10$
28. $4e^x = 91$
29. $7 - 2e^x = 5$
30. $-14 + 3e^x = 11$
31. $e^{3x} = 12$
32. $e^{2x} = 50$
33. $500e^{-x} = 300$
34. $1000e^{-4x} = 75$
35. $e^{2x} - 4e^x - 5 = 0$
36. $e^{2x} - 5e^x + 6 = 0$
37. $20(100 - e^{x/2}) = 500$
38. $\dfrac{400}{1 + e^{-x}} = 350$
39. $10^x = 42$
40. $10^x = 570$
41. $3^{2x} = 80$
42. $6^{5x} = 3000$
43. $5^{-t/2} = 0.20$
44. $4^{-3t} = 0.10$
45. $2^{3-x} = 565$
46. $\left(1 + \dfrac{0.10}{12}\right)^{12t} = 2$

In Exercises 47–50, use a graphing utility to graph the function and approximate its zero accurate to three decimal places.

47. $g(x) = 6e^{1-x} - 25$
48. $f(x) = 3e^{3x/2} - 962$
49. $g(t) = e^{0.09t} - 3$
50. $h(t) = e^{0.125t} - 8$

In Exercises 51–54, solve the exponential equation. Round the result to three decimal places.

51. $8(10^{3x}) = 12$
52. $3(5^{x-1}) = 21$
53. $\left(1 + \dfrac{0.065}{365}\right)^{365t} = 4$
54. $\dfrac{3000}{2 + e^{2x}} = 2$

In Exercises 55–70, solve the logarithmic equation algebraically. Round the result to three decimal places.

55. $\ln x = -3$
56. $\ln x = 2$
57. $\ln 2x = 2.4$
58. $3 \ln 5x = 10$
59. $\ln\sqrt{x + 2} = 1$
60. $\ln(x + 1)^2 = 2$
61. $\log_{10}(z - 3) = 2$
62. $\log_{10} x^2 = 6$
63. $\ln x + \ln(x - 2) = 1$
64. $\ln x + \ln(x + 3) = 1$
65. $\log_{10}(x + 4) - \log_{10} x = \log_{10}(x + 2)$
66. $\log_4 x - \log_4(x - 1) = \frac{1}{2}$
67. $\log_3 x + \log_3(x^2 - 8) = \log_3 8x$
68. $\log_2 x + \log_2(x + 2) = \log_2(x + 6)$
69. $\ln(x + 5) = \ln(x - 1) - \ln(x + 1)$
70. $\ln(x + 1) - \ln(x - 2) = \ln x^2$

In Exercises 71–76, solve the logarithmic equation. Round the result to three decimal places.

71. $6 \log_3(0.5x) = 11$
72. $5 \log_{10}(x - 2) = 11$
73. $2 \ln x = 7$
74. $\ln 4x = 1$
75. $\ln x + \ln(x^2 + 1) = 8$
76. $\log_{10} 8x - \log_{10}(1 + \sqrt{x}) = 2$

56. ***Climb Rate*** The time t, in minutes, for a small plane to climb to an altitude of h feet is given by

$$t = 50 \log_{10} \frac{18{,}000}{18{,}000 - h}$$

where 18,000 feet is the plane's absolute ceiling.

(a) Determine the domain of the function appropriate for the context of the problem.

(b) Use a graphing utility to graph the time function and identify any asymptotes.

(c) As the plane approaches its absolute ceiling, what can be said about the time required to further increase its altitude?

(d) Find the time for the plane to climb to an altitude of 4000 feet.

In Exercises 57–62, solve the exponential equation. Round your result to three decimal places.

57. $e^x = 12$

58. $e^{3x} = 25$

59. $3e^{-5x} = 132$

60. $14e^{3x+2} = 560$

61. $e^{2x} - 7e^x + 10 = 0$

62. $e^{2x} - 6e^x + 8 = 0$

In Exercises 63–68, solve the logarithmic equation. Round the result to three decimal places.

63. $\ln 3x = 8.2$

64. $2 \ln 4x = 15$

65. $\ln x - \ln 3 = 2$

66. $\ln\sqrt{x+1} = 2$

67. $\log(x - 1) = \log(x - 2) - \log(x + 2)$

68. $\log(1 - x) = -1$

In Exercises 69–72, use a graphing utility to solve the equation. Round the result to two decimal places.

69. $2^{0.6x} - 3x = 0$

70. $25e^{-0.3x} = 12$

71. $2 \ln(x + 3) + 3x = 8$

72. $6 \log_{10}(x^2 + 1) - x = 0$

In Exercises 73 and 74, find the exponential function $y = ae^{bx}$ that passes through the points.

73. $(0, 2), (4, 3)$

74. $\left(0, \frac{1}{2}\right), (5, 5)$

75. ***Demand Function*** The demand equation for a certain product is given by

$p = 500 - 0.5e^{0.004x}$.

Find the demand x for a price of (a) $p = \$450$ and (b) $p = \$400$.

76. ***Typing Speed*** In a typing class, the average number of words per minute typed after t weeks of lessons was found to be

$$N = \frac{157}{1 + 5.4e^{-0.12t}}.$$

Find the time necessary to type (a) 50 words per minute and (b) 75 words per minute.

77. ***Compound Interest*** A deposit of \$10,000 is made in a savings account for which the interest is compounded continuously. The balance will double in 5 years.

(a) What is the annual interest rate for this account?

(b) Find the balance after 1 year.

78. ***Sound Intensity*** The relationship between the number of decibels β and the intensity of a sound I in watts per square centimeter is given by

$$\beta = 10 \log_{10}\left(\frac{I}{10^{-16}}\right).$$

Determine the intensity of a sound in watts per square centimeter if the decibel level is 125.

79. ***Earthquake Magnitudes*** On the Richter scale, the magnitude R of an earthquake of intensity I is given by

$$R = \log_{10} \frac{I}{I_0}$$

where $I_0 = 1$ is the minimum intensity used for comparison. Find the intensity per unit of area for the following values of R.

(a) $R = 8.4$

(b) $R = 6.85$

(c) $R = 9.1$